

Computer Vision  
and Geometry Lab

# Computer Vision

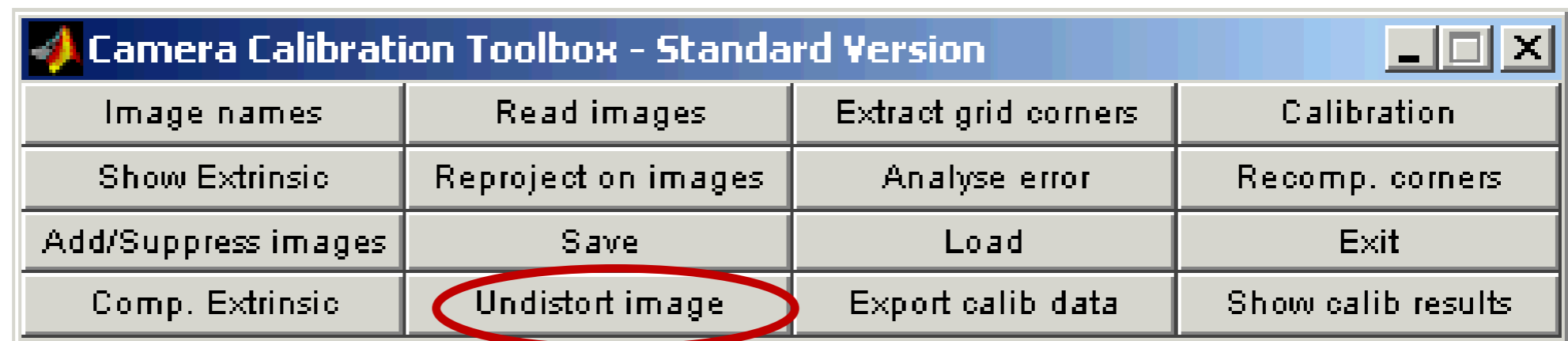
## Exercise Session 2

# Assignment 2

- 4 Tasks:
  - Image capturing
  - Fundamental matrix, eight-point algorithm
  - Essential matrix, eight-point algorithm
  - Camera matrix
- Good reference:  
Multiple View Geometry in computer vision  
(Richard Hartley & Andrew Zisserman)

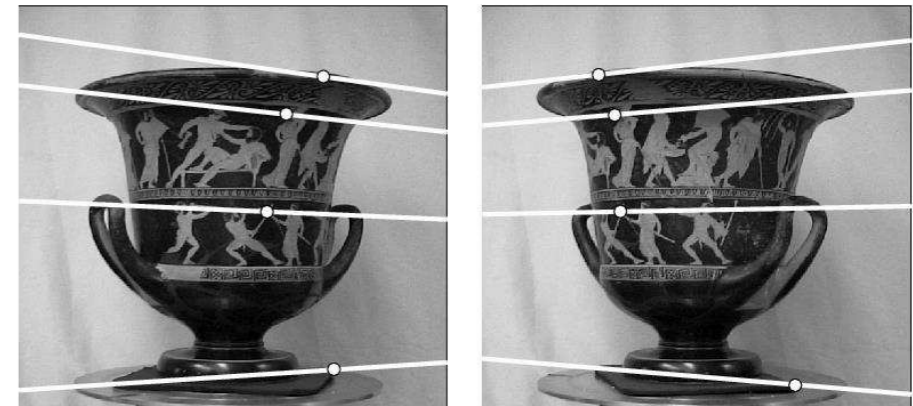
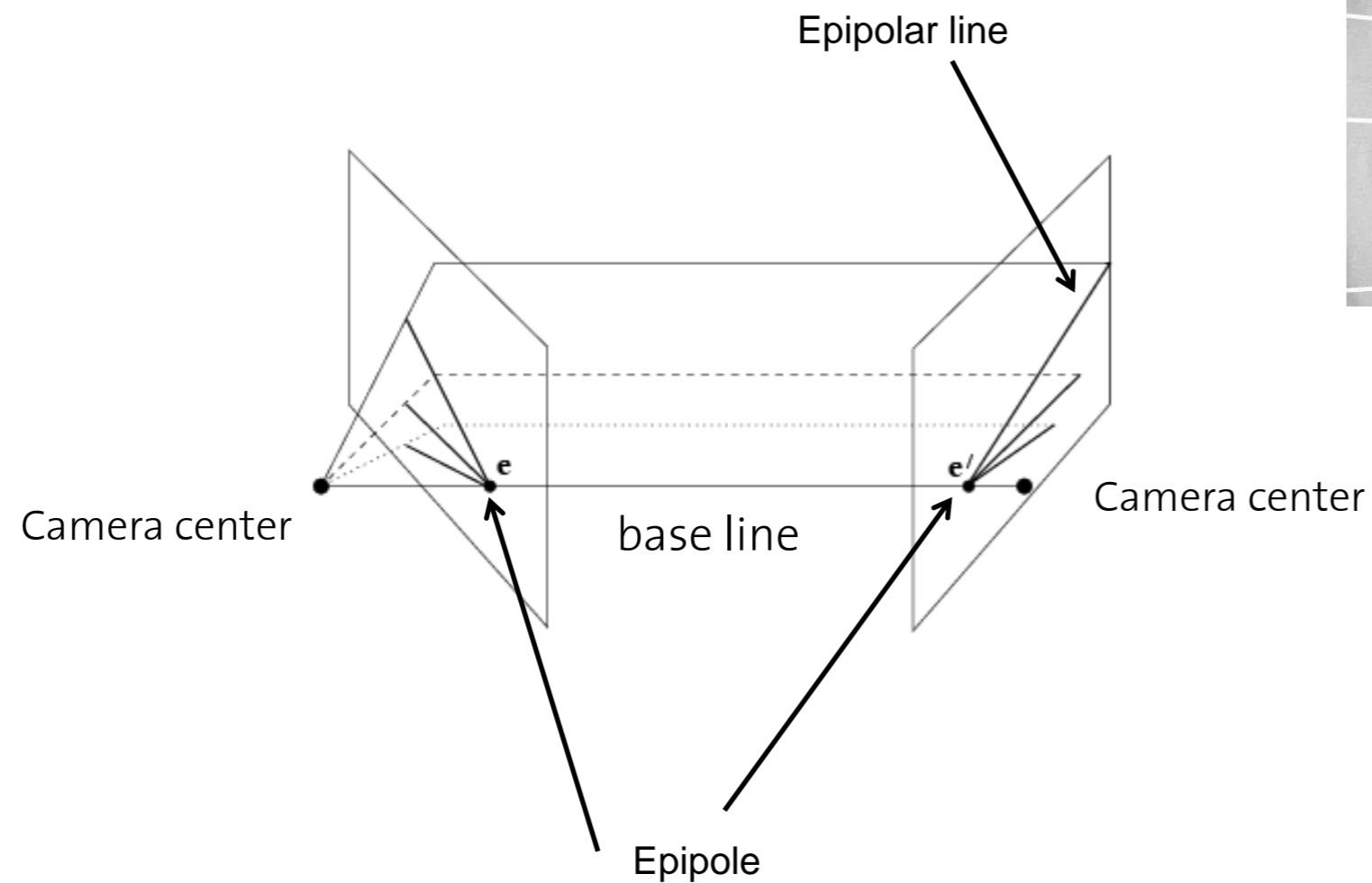
# Image capturing

- Capture two images from same static scene
- The two images should be taken from different viewpoints
- Undistort the images, using Bouget's calibration toolbox



# Fundamental matrix

- Epipolar constraint  $x'^T Fx = 0$



# Fundamental matrix

F is the unique  $3 \times 3$  rank 2 matrix that satisfies  $x'^T F x = 0$  for all  $x \leftrightarrow x'$

- (i) **Transpose:** if F is fundamental matrix for (P,P'), then  $F^T$  is fundamental matrix for (P',P)
- (ii) **Epipolar lines:**  $l' = Fx$  &  $l = F^T x'$
- (iii) **Epipoles:** on all epipolar lines, thus  $e'^T F x = 0, \forall x \Rightarrow e'^T F = 0$ , similarly  $F e = 0$
- (iv) **F** has 7 d.o.f., i.e.  $3 \times 3 - 1$  (homogeneous) - 1 (rank 2)

# Eight-point algorithm

- Epipolar constraint  $x'^T F x = 0$

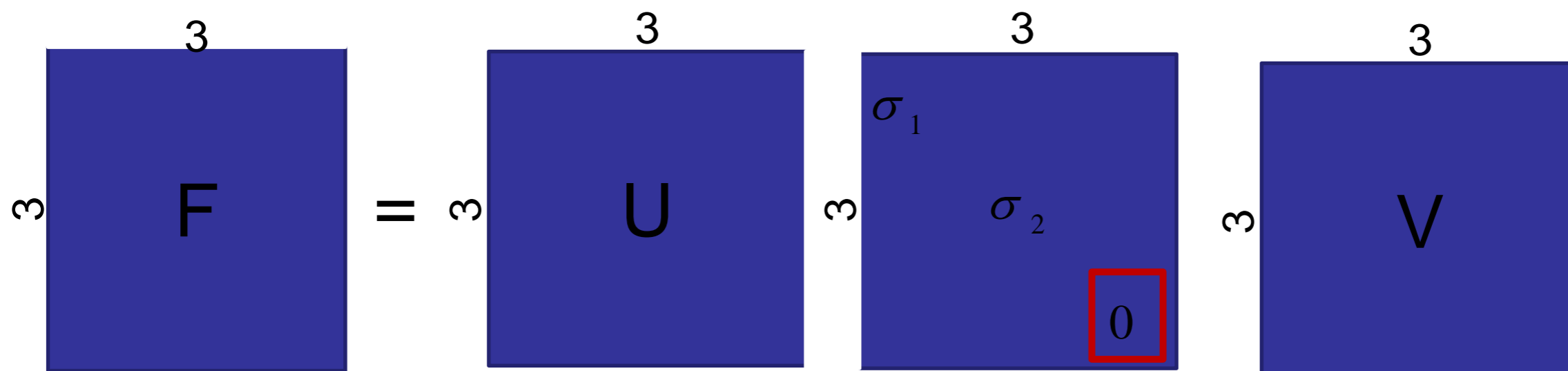
$$\begin{array}{l}
 x = (x, y, 1)^T \quad x' = (x', y', 1)^T \\
 (x' \quad y' \quad 1) \begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = 0 \quad \Rightarrow \quad (x'_1 \ x_1 \quad x'_1 \ y_1 \quad x'_1 \quad y'_1 \ x_1 \quad y'_1 \ y_1 \quad y'_1 \quad x_1 \quad y_1 \quad 1) \begin{pmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{pmatrix} = 0
 \end{array}$$

$$\left( \begin{array}{cccccccc}
 x'_1 \ x_1 & x'_1 \ y_1 & x'_1 & y'_1 \ x_1 & y'_1 \ y_1 & y'_1 & x_1 & y_1 & 1 \\
 x'_n \ x_n & x'_n \ y_n & x'_n & y'_n \ x_n & y'_n \ y_n & y'_n & x_n & y_n & 1
 \end{array} \right) f = 0$$

- Use SVD to solve for F

# Eight-point algorithm

- Enforce the singularity constraint on  $F$ 
  - Factorize  $F$  using SVD
  - Set the third singular value of  $F$  to zero



# Essential matrix (calibrated cameras)

- Essential matrix (5 d.o.f)

$$nx' E nx = 0 \quad E = [t]_{\times} R \quad [t]_{\times} = \begin{bmatrix} 0 & t_z & -t_y \\ -t_z & 0 & t_x \\ t_y & -t_x & 0 \end{bmatrix}$$

- $nx' \leftrightarrow nx$  are the normalized image coordinates

$$nx = K^{-1} x \quad nx' = K^{-1} x'$$

- $K^{-1}$  inverse camera calibration matrix
- Linear solution for  $E$  using 8 point correspondences
- Enforce the property of  $E$  that the first two singular values are equal and the third is zero
- Compare  $F$  with  $K^{-T} E K^{-1}$



# Essential matrix (calibrated cameras)

- Enforce the property of  $E$  that the first two singular values are equal and the third is zero.
  - Factorize  $E$  using SVD, where  $S$  is the diagonal matrix with the singular values,  $S = \text{diag}(r, s, t)$ .
  - Replace  $S$  with  $\text{diag}((r + s)/2, (r + s)/2, 0)$ .

# Essential Matrix

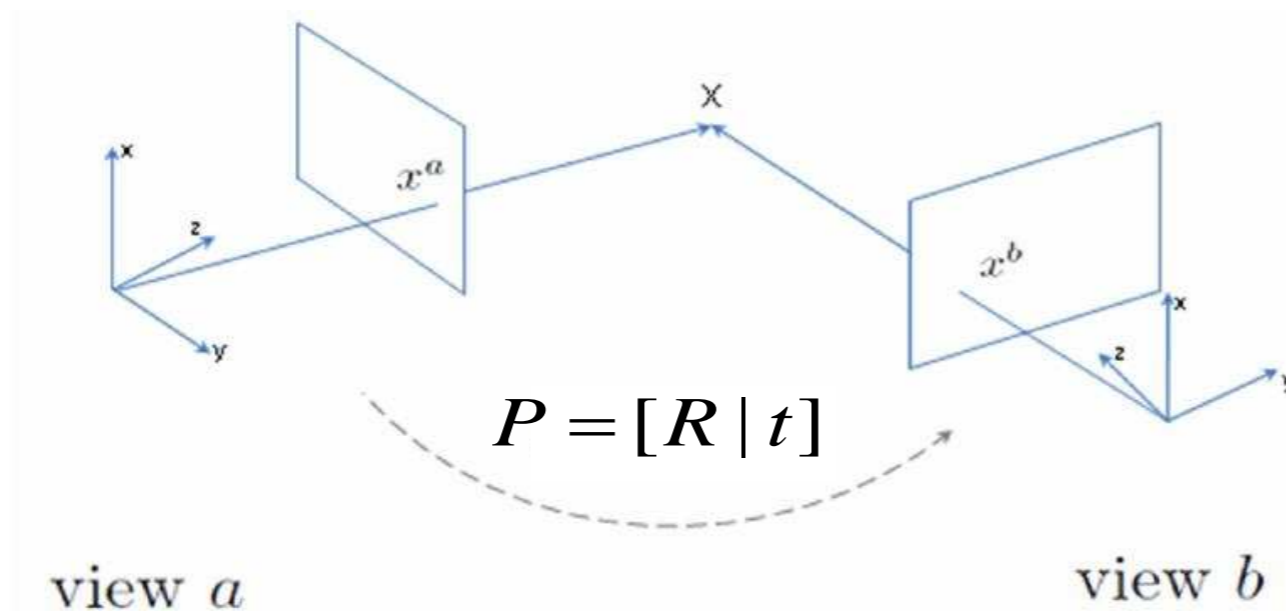
- Decompose  $E = [t]_{\times} R$ 
  - Translation  $t$  is the left null-vector of  $E$  (ie last column of  $U$  if  $E = UDV^T$ )
    - The length of  $t$  is unknown and can be set to 1
  - Rotation matrix is obtained by decomposing

$$E = USV^T$$

$$R_1 = UWV^T, R_2 = UW^T V^T \text{ with } W = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

For RHS coordinate system:

$$\det(R) = 1$$



# Essential Matrix

- We obtain four possible solutions

$$P_1 = R_1[I_{3 \times 3}|t], P_2 = R_1[I_{3 \times 3}|-t], P_3 = R_2[I_{3 \times 3}|t], P_4 = R_2[I_{3 \times 3}|-t]$$

- Correct solution -> triangulated points in front of both cameras

