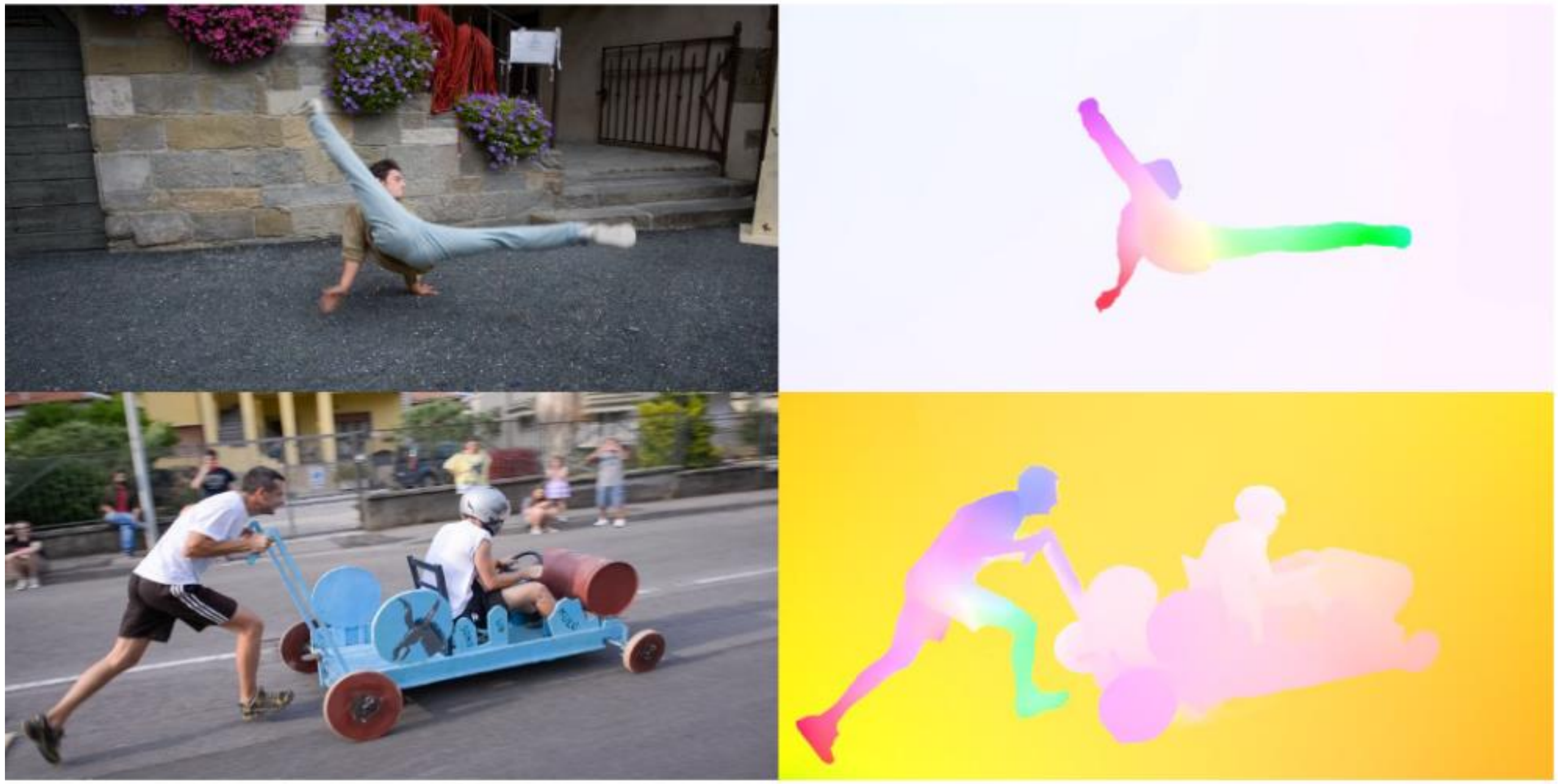
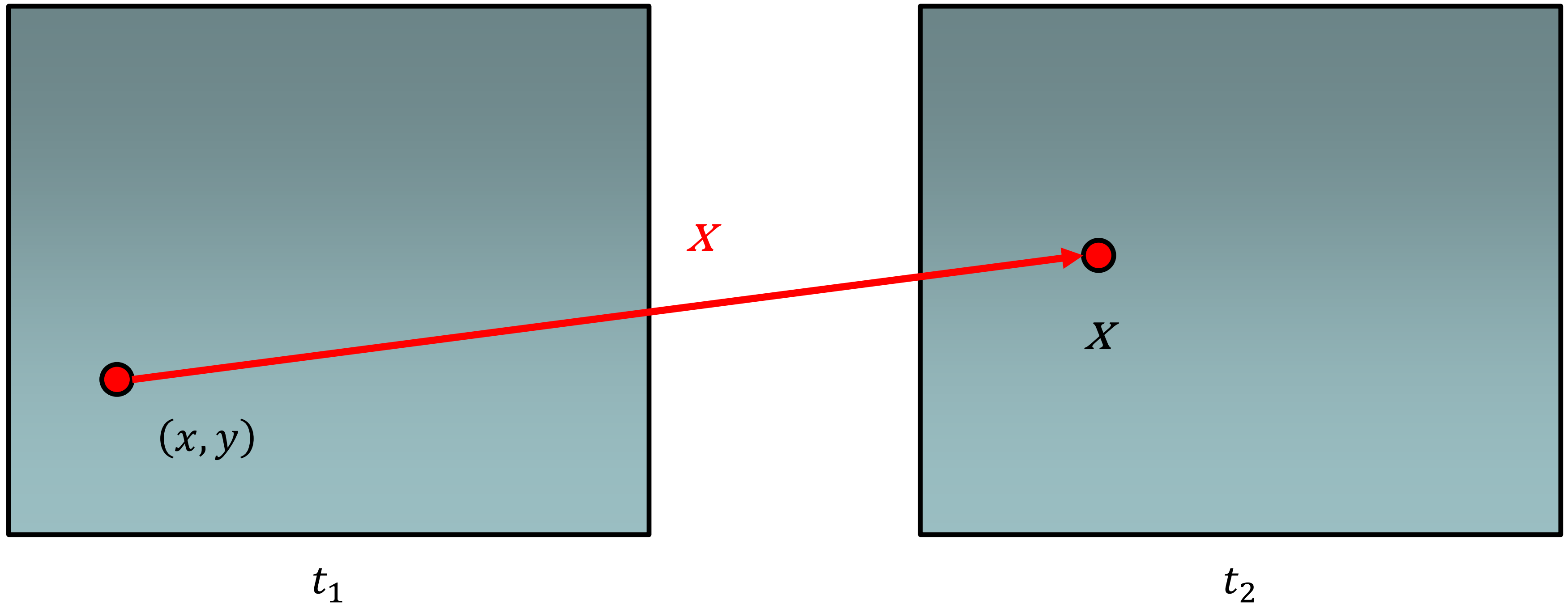
Tutorial 5: Optical Flow



Optical Flow

- What is optical flow?
  - apparent motion of brightness patterns
  - ideally: projection of a 3D motion into the 2D image plane



Optical Flow

- Dense: estimated for each pixel

KITTI and MPI Sintel Flow datasets

Applications

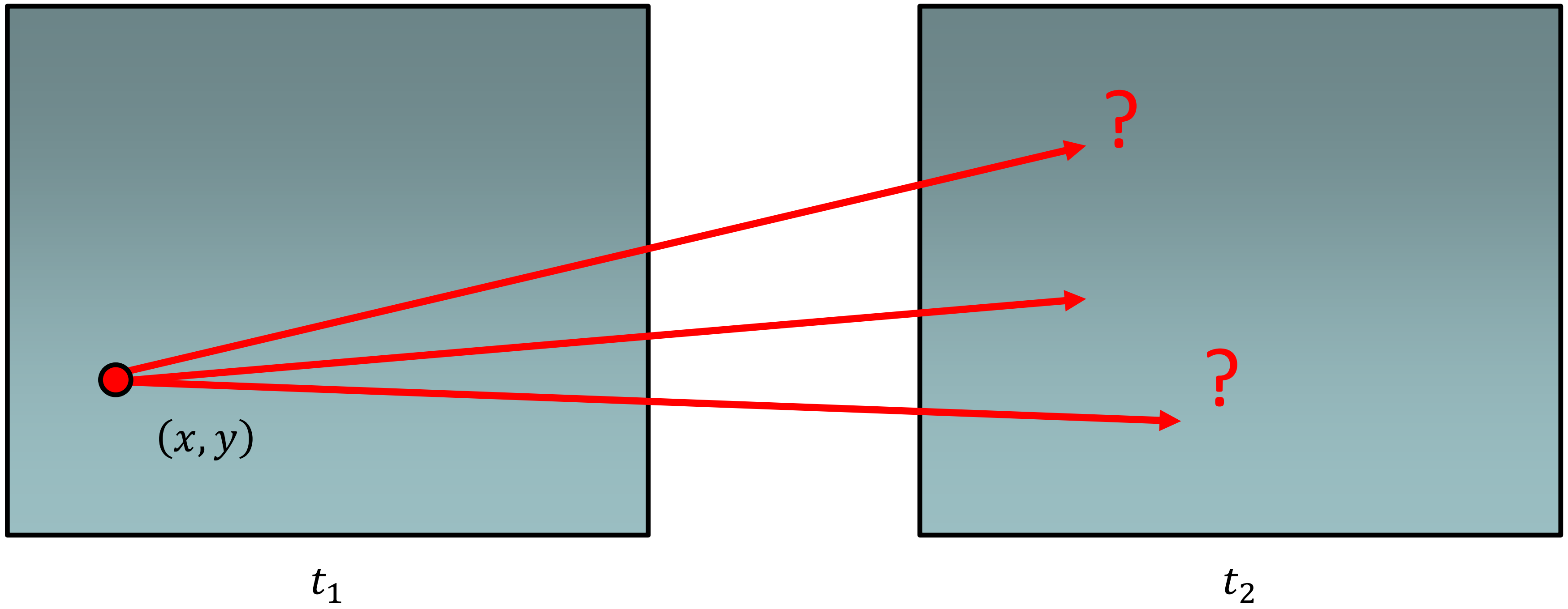
- Object Segmentation and Tracking

Applications

- Frame Interpolation and Slow Motion

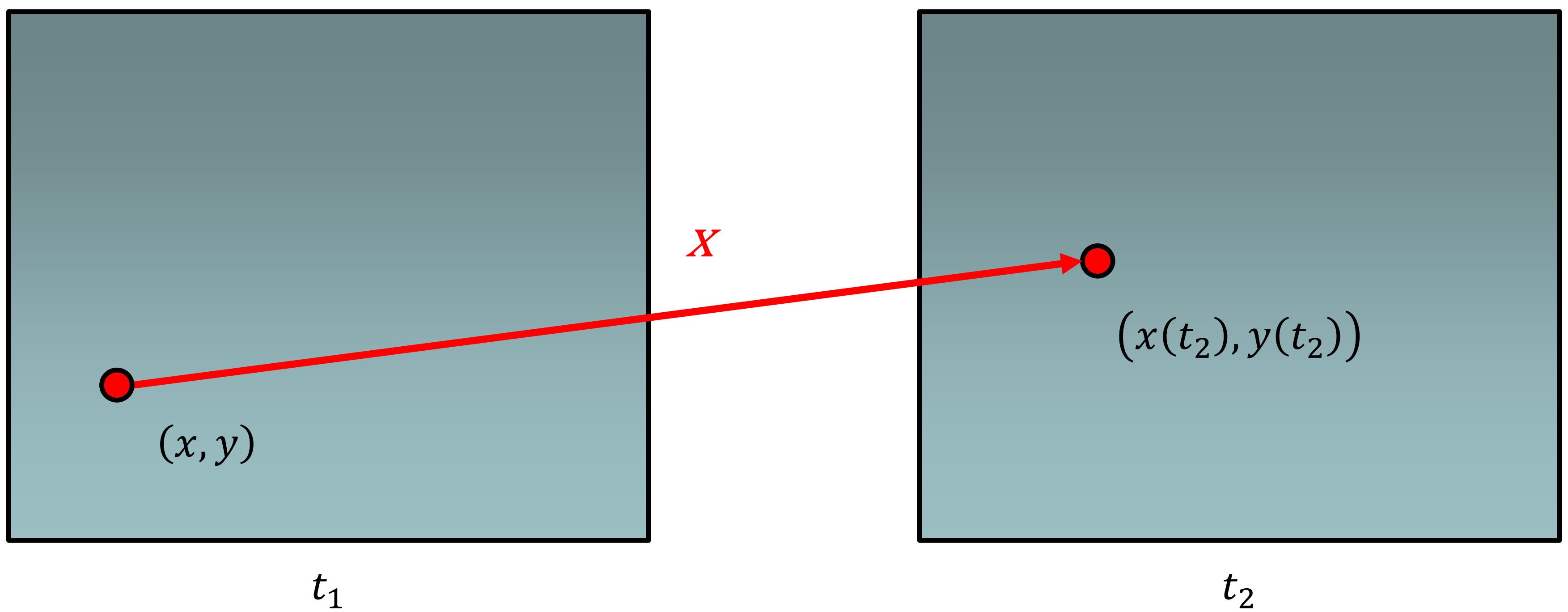
Optical Flow Estimation

How to estimate optical flow given two frames?



Optical Flow Estimation

How to estimate optical flow given two frames?

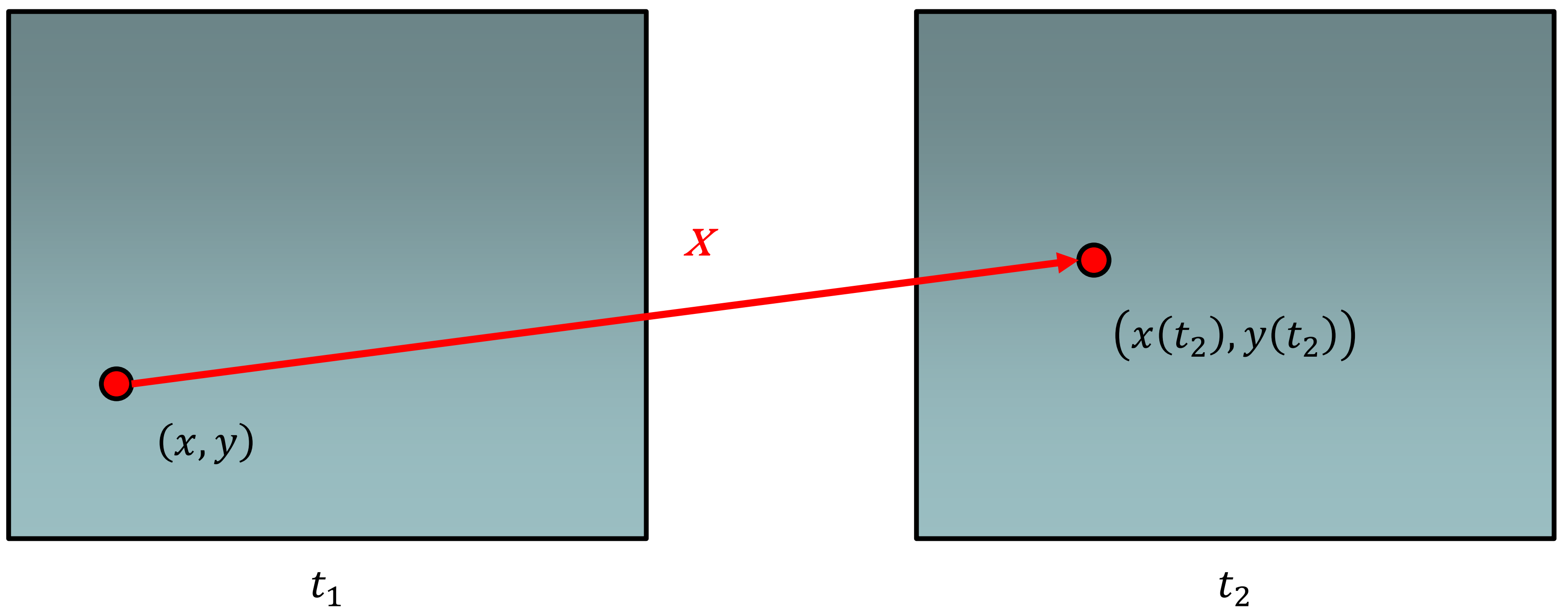


Optical Flow Estimation

How to estimate optical flow given two frames?

Assumption 1:

brightness of the point will remain the same



$$I(x(t), y(t), t) = C$$

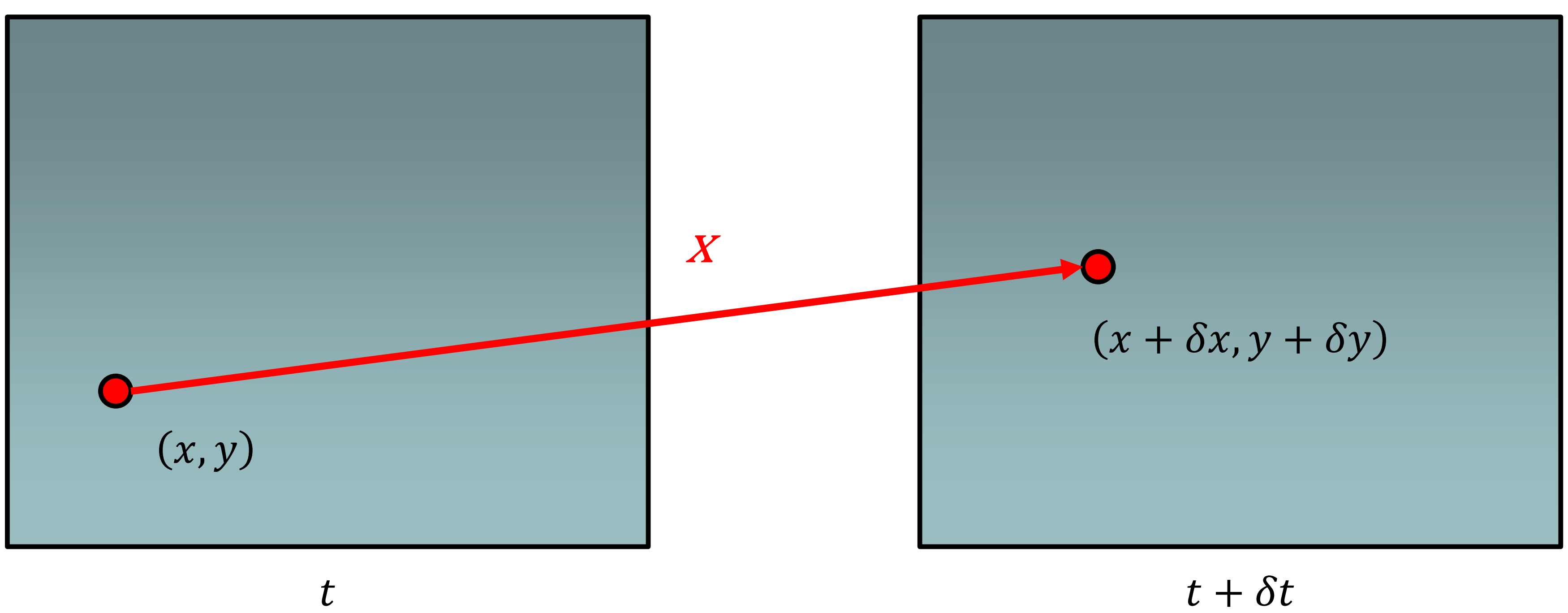
Brightness Constancy

Optical Flow Estimation

How to estimate optical flow given two frames?

Assumption 2:

Small motion



For very a small space-time step:

$$I(x, y, t) = I(x + \delta x, y + \delta y, t + \delta t)$$

Brightness Constancy

$$I(x, y, t) = I(x + \delta x, y + \delta y, t + \delta t)$$

Brightness Constancy

$$I(x, y, t) = I(x + \delta x, y + \delta y, t + \delta t)$$

Taylor expansion

$$I(x, y, t) \approx I(x, y, t) + \frac{\partial I}{\partial x}\delta x + \frac{\partial I}{\partial y}\delta y + \frac{\partial I}{\partial t}\delta t$$

Brightness Constancy

$$I(x, y, t) = I(x + \delta x, y + \delta y, t + \delta t)$$

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Brightness Constancy

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$$\cancel{I(x,y,t)} \approx \cancel{I(x,y,t)} + \frac{\partial I}{\partial x}\delta x + \frac{\partial I}{\partial y}\delta y + \frac{\partial I}{\partial t}\delta t$$

$$\frac{\partial I}{\partial x}\frac{dx}{dt} + \frac{\partial I}{\partial y}\frac{dy}{dt} + \frac{\partial I}{\partial t} \approx 0$$

Brightness Constancy

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$$I_x \cdot u + I_y \cdot v + I_t \approx 0$$

Brightness Constancy

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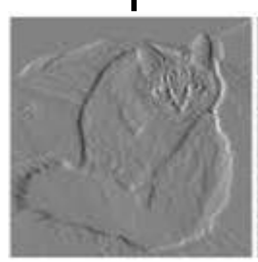
Optical Flow

Brightness Constancy

$$\frac{\partial I}{\partial x}\frac{dx}{dt} + \frac{\partial I}{\partial y}\frac{dy}{dt} + \frac{\partial I}{\partial t} \approx 0$$

Spatial partial derivatives $I_x \cdot u + I_y \cdot v + I_t \approx 0$



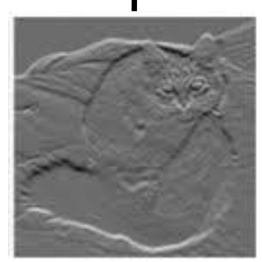


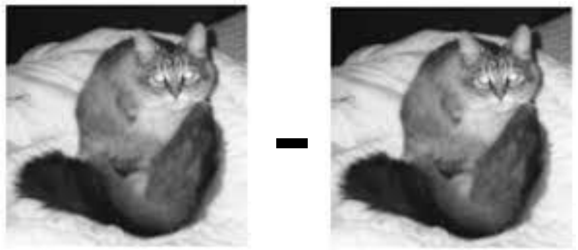
Image gradient along x / y direction
e.g. with Sobel Filter

Brightness Constancy

$$\frac{\partial I}{\partial x}\frac{dx}{dt} + \frac{\partial I}{\partial y}\frac{dy}{dt} + \frac{\partial I}{\partial t} \approx 0$$

$$I_x \cdot u + I_y \cdot v + I_t \approx 0$$

Temporal partial derivatives



Difference between two frames

Brightness Constancy

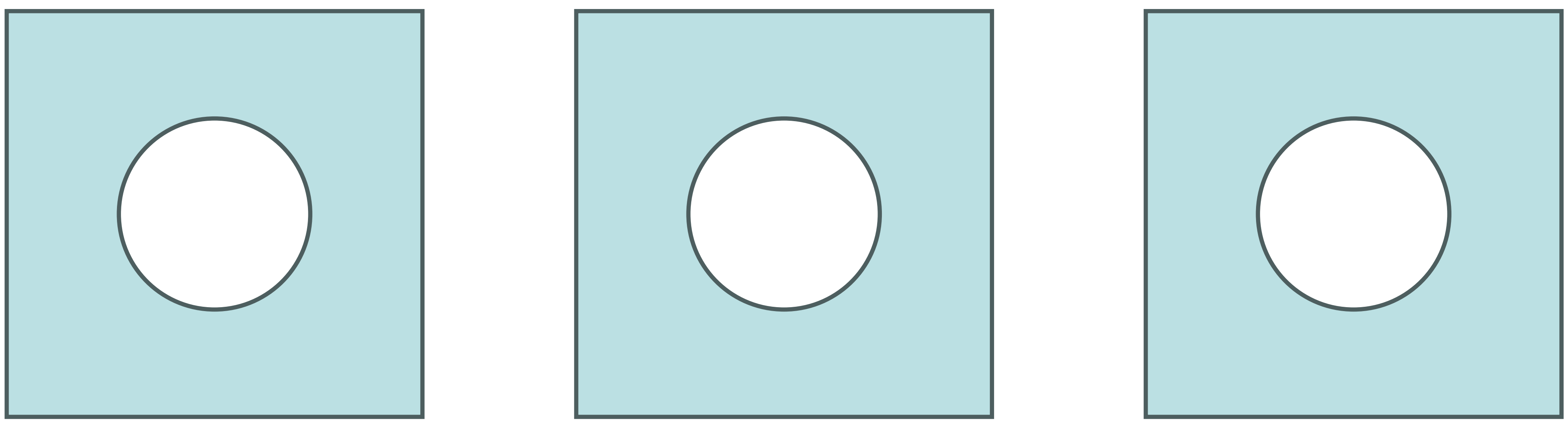
$$\frac{\partial I}{\partial x}\frac{dx}{dt} + \frac{\partial I}{\partial y}\frac{dy}{dt} + \frac{\partial I}{\partial t} \approx 0$$

$$I_x \cdot u + I_y \cdot v + I_t \approx 0$$

One equation, two unknowns

Aperture Problem

- The local motion is inherently ambiguous with respect to the global motion
- 1 degree of freedom along the line



Aperture Problem

- The local motion is inherently ambiguous with respect to the global motion
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Brightness Constancy

$$\frac{\partial I}{\partial x}\frac{dx}{dt} + \frac{\partial I}{\partial y}\frac{dy}{dt} + \frac{\partial I}{\partial t} \approx 0$$

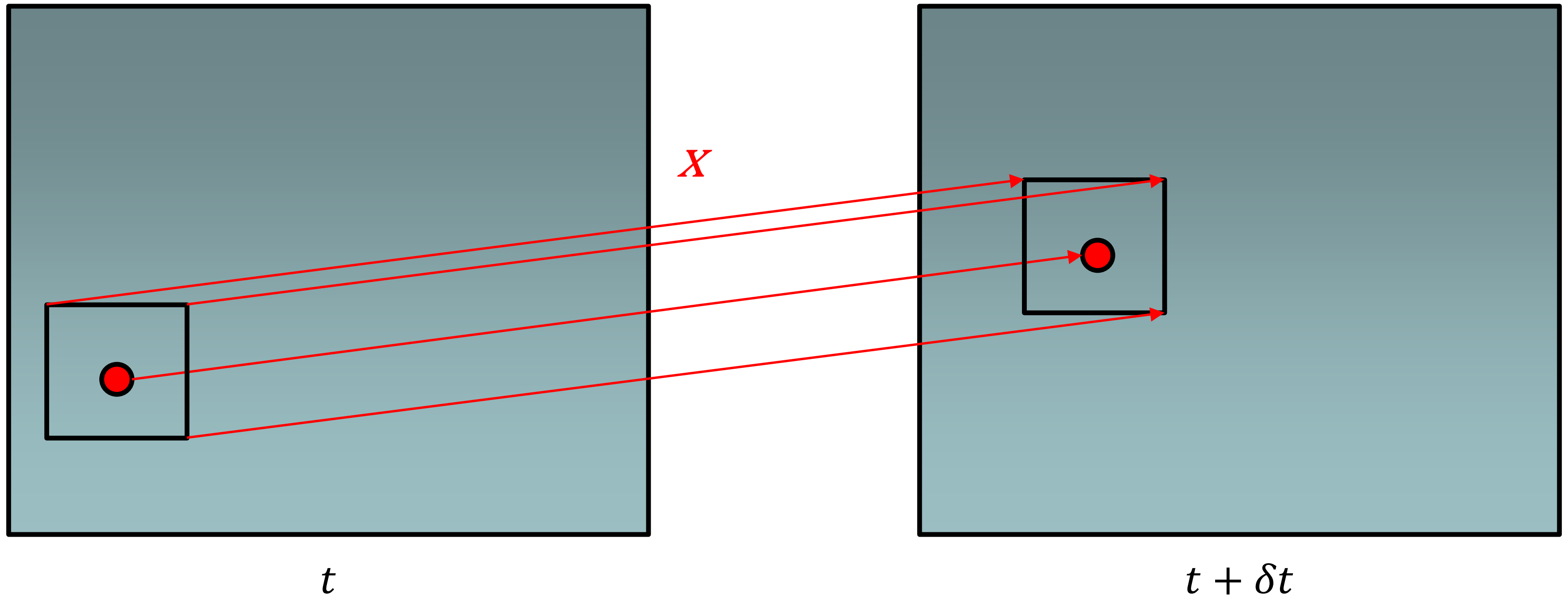
$$I_x \cdot u + I_y \cdot v + I_t \approx 0$$

One equation, two unknowns

→ We need more constraints (equations)

Spatial Coherency

- Assume the same flow for all pixels within a patch.
  = Flow is locally smooth



Spatial Coherency

- Assume the same flow for all pixels within a patch.

$$\begin{bmatrix} I_x(\mathrm{p}_1) & I_y(\mathrm{p}_1) \\ I_x(\mathrm{p}_2) & I_y(\mathrm{p}_2) \\ \vdots & \vdots \\ I_x(\mathrm{p}_{25}) & I_y(\mathrm{p}_{25}) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} I_t(\mathrm{p}_1) \\ I_t(\mathrm{p}_2) \\ \vdots \\ I_t(\mathrm{p}_{25}) \end{bmatrix}$$

5x5 patch
= 25 equations

Spatial Coherency

- Assume the same flow for all pixels within a patch.

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- Estimate the optical flow by minimizing the error over a patch ➔ solve the linear system

Spatial Coherency

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- Estimate the optical flow by minimizing the error over a patch ➔ solve the linear system
- Solution given by

Spatial Coherency

- Assume the same flow for all pixels within a patch.

$$\begin{bmatrix} I_x(\mathrm{p}_1) & I_y(\mathrm{p}_1) \\ I_x(\mathrm{p}_2) & I_y(\mathrm{p}_2) \\ \vdots & \vdots \\ I_x(\mathrm{p}_{25}) & I_y(\mathrm{p}_{25}) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} I_t(\mathrm{p}_1) \\ I_t(\mathrm{p}_2) \\ \vdots \\ I_t(\mathrm{p}_{25}) \end{bmatrix}$$

- Estimate the optical flow by minimizing the error over a patch ➔ solve the linear system
- Solution given by Lukas-Kanade Algorithm

Part A. Lucas-Kanade Algorithm

Step 1. Compute partial derivatives.

$$\begin{bmatrix} I_x(\mathrm{p}_1) & I_y(\mathrm{p}_1) \\ I_x(\mathrm{p}_2) & I_y(\mathrm{p}_2) \\ \vdots & \vdots \\ I_x(\mathrm{p}_{25}) & I_y(\mathrm{p}_{25}) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} I_t(\mathrm{p}_1) \\ I_t(\mathrm{p}_2) \\ \vdots \\ I_t(\mathrm{p}_{25}) \end{bmatrix}$$

Part A. Lucas-Kanade Algorithm

Step 1. Compute partial derivatives.

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Step 2. Construct and solve the above linear system.

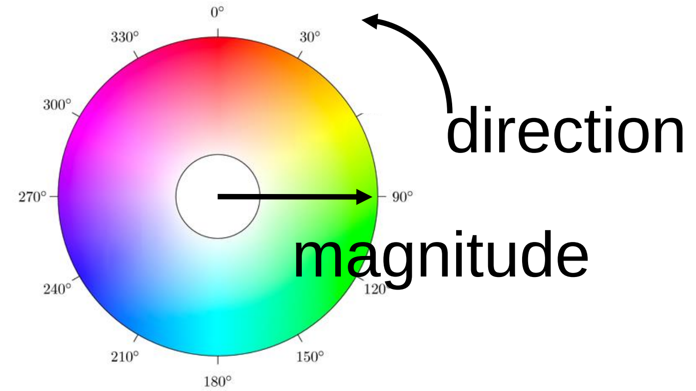
Part A. Lucas-Kanade Algorithm

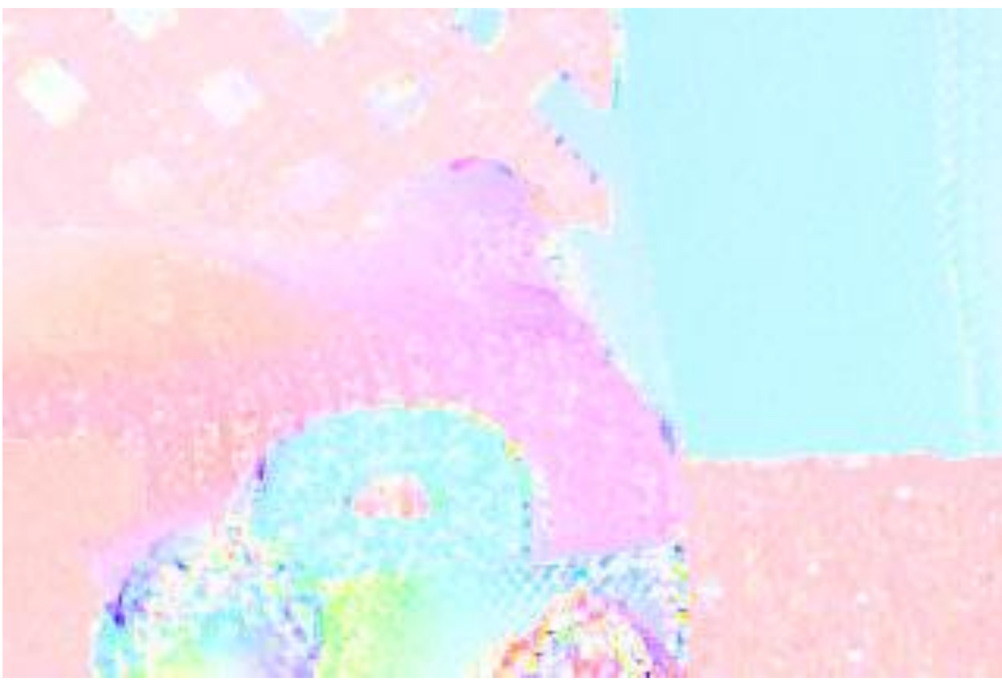
Image 1

Image 2

Part A. Lucas-Kanade Algorithm

Part A. Lucas-Kanade Algorithm



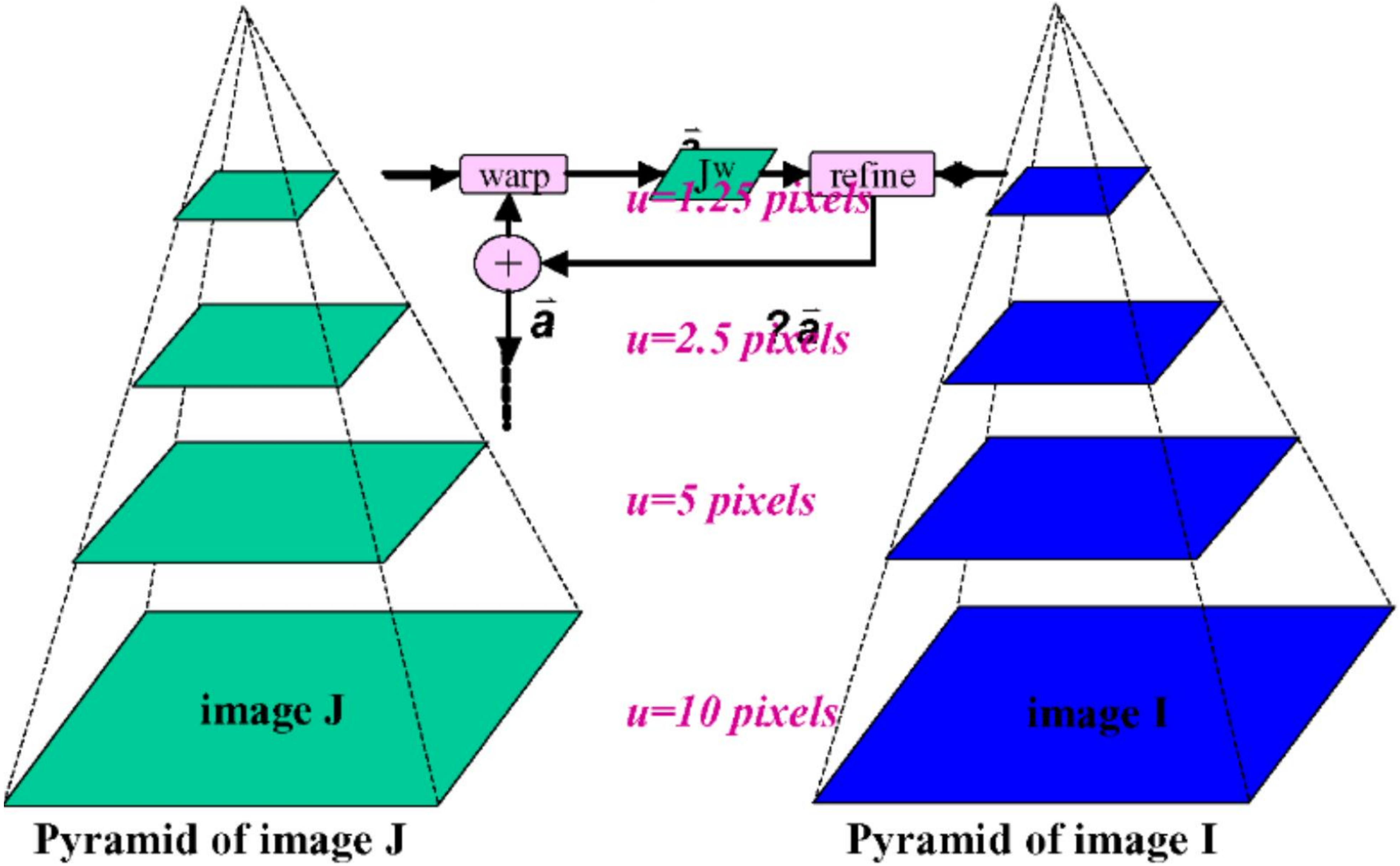


Result

Ground-truth

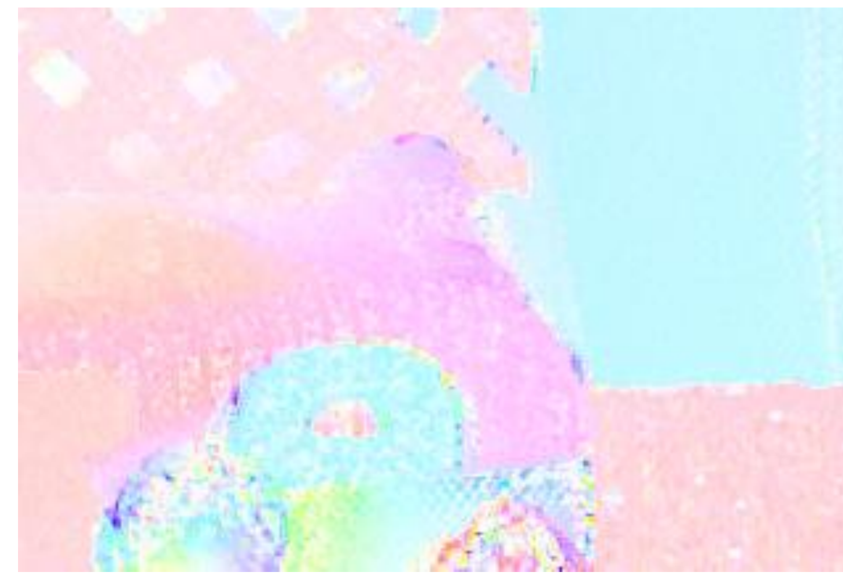
Part B. Lucas-Kanade with Pyramids

$$I_x \cdot u + I_y \cdot v + I_t \approx 0$$ ==> small *u* and *v* ...



Part B. Coarse-to-Fine Estimation

with pyramids



without pyramids

Ground-truth

Part C. Frame Extrapolation

Image 1 and 2

Part C. Frame Extrapolation

Extrapolated frames

Takeaways

- Optical flow with Lucas-Kanade
- Assume brightness constancy + small motion
- Image gradients + temporal difference
- Use image pyramids for larger motions

Exercise

Two options:

- GitHub + jupyter notebooks run locally
  https://github.com/tavisualcomputing/viscomp2023
- Google Colab: Python notebook in the cloud
  https://colab.research.google.com/github/tavisualcomputing/viscomp2023/blob/main/Exercises/W6/W6_exercise.ipynb
- Questions: Moodle forum https://moodle-app2.let.ethz.ch/mod/forum/view.php?id=964720