Lighting and Shading I
Prof. Dr. Markus Gross
Before: nature of light and colors

<table>
<thead>
<tr>
<th>Wave Type</th>
<th>Wavelength (λ) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>γ rays</td>
<td>10^{-16}</td>
</tr>
<tr>
<td>X rays</td>
<td>10^{-14}</td>
</tr>
<tr>
<td>UV</td>
<td>10^{-12}</td>
</tr>
<tr>
<td>IR</td>
<td>10^{-10}</td>
</tr>
<tr>
<td>Microwave</td>
<td>10^{-8}</td>
</tr>
<tr>
<td>FM Radio</td>
<td>10^{-6}</td>
</tr>
<tr>
<td>AM Radio</td>
<td>10^{-4}</td>
</tr>
<tr>
<td>Long radio</td>
<td>10^{0}</td>
</tr>
<tr>
<td></td>
<td>10^2</td>
</tr>
<tr>
<td></td>
<td>10^4</td>
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<td></td>
<td>10^6</td>
</tr>
<tr>
<td></td>
<td>10^8</td>
</tr>
</tbody>
</table>

Visible spectrum:

400 450 500 550 600 650 700 750
Before: nature of light and colors

• Light can be a mixture of many wavelengths

• Spectral power distribution (SPD)
  - \( P(\lambda) = \text{intensity at wavelength } \lambda \)
  - intensity as a function of wavelength

• We perceive these distributions as colors
Before: representing colors

- Unit cube with R,G,B basis vectors

![Diagram showing the unit cube with R,G,B basis vectors and colors associated with each vector.]

- Red (1,0,0)
- Yellow (1,1,0)
- Green (0,1,0)
- White (1,1,1)
- Black (0,0,0)
- Magenta (1,0,1)
- Blue (0,0,1)
- Cyan (0,1,1)
- Magenta (1,0,1)
- Green (0,1,0)
- White (1,1,1)
Before: we need material models

- Interaction of light with geometry
Measuring Light

• How do we measure light

Measuring = Counting photons
Radiometry

- Studies the measurement of electromagnetic radiation, including visible light
Basic Definitions

• Angle: \( \theta = \frac{l}{r} \)
  - circle: \( 2\pi \) radians

• Solid angle: \( \Omega = \frac{A}{r^2} \)
  - sphere: \( 4\pi \) steradians
Basic Definitions

- Direction
  - point on the unit sphere
  - parameterized by two angles

\[ \vec{\omega} = \left( \theta, \phi \right) \]

- \( \vec{\omega}_x = \sin \theta \cos \phi \)
- \( \vec{\omega}_y = \sin \theta \sin \phi \)
- \( \vec{\omega}_z = \cos \theta \)

\[ \text{latitude} = \frac{90}{\pi} (\pi - \theta) \]
\[ \text{longitude} = \frac{90}{\pi} \phi \]
Basic Definitions

- Differential Solid Angle

\[ dA = (rd\theta)(r \sin \theta d\phi) \]
\[ d\mathbf{\Omega} = \frac{dA}{r^2} = \sin \theta d\theta d\phi \]

\[ \Omega = \int_{S^2} d\mathbf{\Omega} = \int_0^{2\pi} \int_0^\pi \sin \theta d\theta d\phi = 4\pi \]
Basic Definitions

- Assume light consists of photons with
  - \( \mathbf{x} \): Position
  - \( \hat{\omega} \): Direction of motion
  - \( \lambda \): Wavelength

- Each photon has an energy of: \( \frac{hc}{\lambda} \)
  - \( h \approx 6.63 \cdot 10^{-34} \text{ m}^2 \cdot \text{kg/s} \): Planck's constant
  - \( c = 299,792,458 \text{ m/s} \): speed of light in vacuum
  - Unit of energy, Joule: \([ J = \text{kg} \cdot \text{m}^2 / \text{s}^2 ]\)
Radiometry

• Basic quantities
  – flux $\Phi$
  – irradiance $E$
  – radiosity $B$
  – intensity $I$
  – radiance $L$
Radiometry

- **Flux (radiant flux, power)**
  - total amount of energy passing through surface or space per unit time

\[ \Phi(A) \quad \left[ \frac{J}{s} = W \right] \]

- examples:
  - number of photons hitting a wall per second
  - number of photons leaving a lightbulb per second
Radiometry

• Irradiance
  – flux per unit area *arriving* at a surface = area density of flux

\[ E(x) = \frac{d\Phi(A)}{dA(x)} \left[ \frac{W}{m^2} \right] \]

– example:
  • number of photons hitting a small patch of a wall per second, divided by the size of the patch
Radiometry

• Radiosity
  – flux per unit area *leaving* a surface = area density of flux

\[
B(x) = \frac{d\Phi(A)}{dA(x)} \left[ \frac{W}{m^2} \right]
\]

– example:
  • number of photons reflecting off a small patch of a wall per second, divided by the size of the patch
Radiometry

- Irradiance
  - Lambert's Cosine Law

\[ E = \frac{\Phi}{A} \]

\[ E = \frac{\Phi}{A / \cos \theta} = \frac{\Phi}{A} \cos \theta \]
Radiometry

- Irradiance
  - Lambert's Cosine Law

\[ E = \frac{\Phi}{A} \cdot \frac{1}{\cos \theta} = \frac{\Phi}{A} \cdot \cos \theta \]
Radiometry

- Radiant intensity
  - Power (flux) per solid angle = directional density of flux

  \[ I(\vec{\omega}) = \frac{d\Phi}{d\omega} \left[ \frac{W}{sr} \right] \]

  \[ \Phi = \int_{S^2} I(\vec{\omega})d\vec{\omega} \]

- example:
  - power per unit solid angle emanating from a point source

  \[ \Phi = 4\pi I \quad \text{isotropic point source} \]
Radiometry

- Radiance
  - intensity per unit area = flux density per unit solid angle, per perpendicular unit area

\[ L(x, \bar{\omega}) = \frac{dI(\bar{\omega})}{dA(x)} = \frac{d^2 \Phi(A)}{d\bar{\omega} dA^\perp(x, \bar{\omega})} = \frac{d^2 \Phi(A)}{d\bar{\omega} dA(x) \cos \theta} \left[ \frac{W}{m^2 \text{sr}} \right] \]

  - most fundamental for raytracing
  - remains constant along a ray
Other radiometric quantities can be expressed in terms of radiance

- Irradiance: \[ L(x, \omega) = \frac{d^2 \Phi(A)}{\cos \theta dA(x) d\omega} \quad E(x) = \frac{d\Phi(A)}{dA(x)} \]

\[ L(x, \omega) = \frac{dE(x)}{\cos \theta d\omega} \]

\[ L(x, \omega) \cos \theta d\omega = dE(x) \]

\[ \int_{H^2} L(x, \omega) \cos \theta d\omega = E(x) \]

- Integrate radiance over the hemisphere
- Same for radiosity
Radiometry

• Other radiometric quantities can be expressed in terms of radiance
  – Flux:
    \[ E(x) = \frac{d\Phi(A)}{dA(x)} \]
    \[ \int_A E(x) dA(x) = \Phi(A) \]
    \[ \int_A \int_{H^2} L(x, \omega) \cos \theta d\omega dA(x) = \Phi(A) \]
  – Integrate irradiance over area
  – Integrate radiance over hemisphere and area
Radiometry

- Basic quantities

<table>
<thead>
<tr>
<th>Flux</th>
<th>( \Phi(A) )</th>
<th>[ \frac{J}{s} = W ]</th>
</tr>
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<tbody>
<tr>
<td>Irradiance</td>
<td>( E(x) = \frac{d\Phi(A)}{dA(x)} )</td>
<td>[ \frac{W}{m^2} ]</td>
</tr>
<tr>
<td>Radiosity</td>
<td>( B(x) = \frac{d\Phi(A)}{dA(x)} )</td>
<td>[ \frac{W}{m^2} ]</td>
</tr>
<tr>
<td>Intensity</td>
<td>( I(\vec{\omega}) = \frac{d\Phi}{d\vec{\omega}} )</td>
<td>[ \frac{W}{sr} ]</td>
</tr>
<tr>
<td>Radiance</td>
<td>( L(x, \vec{\omega}) = \frac{d^2\Phi(A)}{\cos \theta dA(x)d\vec{\omega}} )</td>
<td>[ \frac{W}{m^2 , sr} ]</td>
</tr>
</tbody>
</table>
Reflection Models

- **Bidirectional Reflectance Distribution Function (BRDF)**
BRDF

• **Bidirectional Reflectance Distribution Function**

\[
f_r(x, \omega_i, \omega_r) = \frac{dL_r(x, \omega_r)}{dE_i(x, \omega_i)} = \frac{dL_r(x, \omega_r)}{L_i(x, \omega_i) \cos \theta_i d\omega_i} \quad [1/\text{sr}]
\]

• Differential irradiance due to a cone of directions around \( \omega_i \)
Reflection Equation

• The BRDF provides a relation between incident radiance and differential reflected radiance
• From this we can derive the Reflection Equation

\[ f_r(x, \omega_i, \omega_r) = \frac{dL_r(x, \omega_r)}{L_i(x, \omega_i) \cos \theta_i \, d\omega_i} \]

\[ f_r(x, \omega_i, \omega_r) L_i(x, \omega_i) \cos \theta_i = \frac{dL_r(x, \omega_r)}{d\omega_i} \]

\[ \int_{H^2} f_r(x, \omega_i, \omega_r) L_i(x, \omega_i) \cos \theta_i \, d\omega_i = L_r(x, \omega_r) \]
The Reflection Equation describes a *local illumination* model

- reflected radiance due to incident illumination from all directions

\[ L_r(x, \hat{\omega}_r) = \int_{H^2} f_r(x, \hat{\omega}_i, \hat{\omega}_r) L_i(x, \hat{\omega}_i) \cos \theta_i \, d\hat{\omega}_i \]
Complex Reflections

layered materials

anisotropic reflections

subsurface scattering

volumetric structures
Measuring BRDFs

Matusik et al.: Efficient Isotropic BRDF Measurement, Eurographics Symposium on Rendering 2003
Simpler Reflections

Hendrik Lensch, Efficient Image-Based Appearance Acquisition of Real-World Objects, Ph.D. thesis, 2004
Diffuse Reflection

- For diffuse reflection, the BRDF is a constant:

\[ L_r(x, \tilde{\omega}_r) = \int_{H^2} f_r(x, \tilde{\omega}_i, \tilde{\omega}_r) L_i(x, \tilde{\omega}_i) \cos \theta_i \, d\tilde{\omega}_i \]

\[ L_r(x) = f_r \int_{H^2} L_i(x, \tilde{\omega}_i) \cos \theta_i \, d\tilde{\omega}_i \]

\[ L_r(x) = f_r E_i(x) \]
Simple Models

• Exact computation too slow
• OpenGL uses simplified reflection models
• Phong illumination
Ambient Light

- Scattered by environment
- Coming from all directions
- Reflection independent of
  - Camera position
  - Light position (no light position)
  - Surface orientation
- Reflected intensity: $I = I_a k_a$
Diffuse Reflection

- Directed light $I_p$
- Reflection dependent on
  - orientation of surface
  - light source position
- Independent of
  - camera position (reflected equally in all directions)
- Reflected intensity:
  \[ I = I_p k_d \cos \theta \]
  \[ I = I_p k_d (\mathbf{N} \cdot \mathbf{L}) \]
A Simple Model

• Sum up ambient light and diffuse reflection:

\[ I = I_a k_a + I_p k_d (\mathbf{N} \cdot \mathbf{L}) \]
Attenuation

- Quadratic attenuation due to spatial radiation
  \[ f_{att} = \frac{1}{d_L^2} \]
- A model often used in Graphics (OpenGL)
  \[ f_{att} = \min \left( \frac{1}{c_1 + c_2 d_L + c_3 d_L^2}, 1 \right) \]
- Include attenuation
  \[ I = I_a k_a + f_{att} I_p k_d (\mathbf{N} \cdot \mathbf{L}) \]
Specular Reflection

• Depends on the angle between the reflection and viewing ray
Specular Reflection

- Compute by simple linear algebra

\[ R = N \cos \theta + S \]

\[ R = 2N \cos \theta - L = 2N(N \cdot L) - L \]

\[ \cos \alpha = R \cdot V = (2N(N \cdot L) - L) \cdot V \]
Ambient + Diffuse + Specular
Phong Illumination Model

- Approximates specular reflection by cosine powers

\[ I_\lambda = I_{a\lambda} k_a O_{d\lambda} + f_{att} I_{p\lambda} [k_d O_{d\lambda} (\mathbf{N} \cdot \mathbf{L}) + k_s (\mathbf{R} \cdot \mathbf{V})^n] \]
Extensions

- Specular colors

\[ I_\lambda = I_{a_\lambda} k_a + f_{att} I_{p_\lambda} [k_d(N \cdot L) + k_s(R \cdot V)^n] \]

Material dependent constants
Extensions

- Specular colors

\[ I_\lambda = I_{a\lambda} k_a + f_{att} I_{p\lambda} \left[ k_d (N \cdot L) + k_s (R \cdot V)^n \right] \]

- Halfway vector (faster)

\[ \cos^n \beta = (N \cdot H)^n \quad H = \frac{L + V}{||L + V||} \]

- Multiple light sources

\[ I_\lambda = I_{a\lambda} k_a + \sum_{1 \leq i \leq m} f_{att_i} I_{p\lambda_i} \left[ k_d (N \cdot L_i) + k_s (R_i \cdot V)^n \right] \]
End