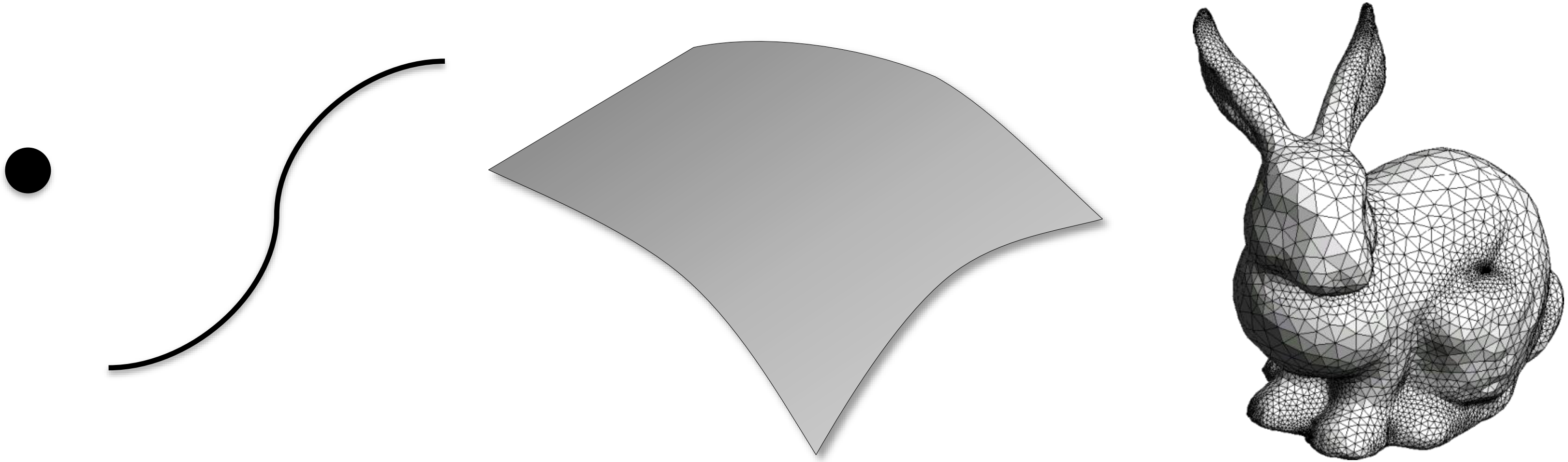
Geometry Processing

Prof. Dr. Markus Gross

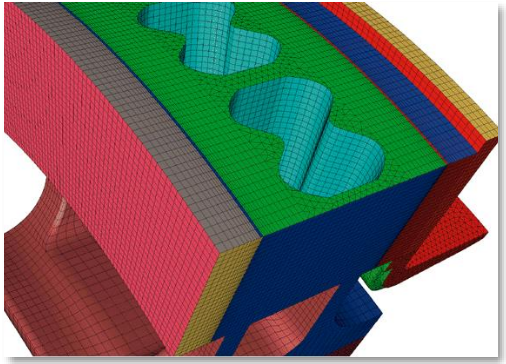
ETH zürich

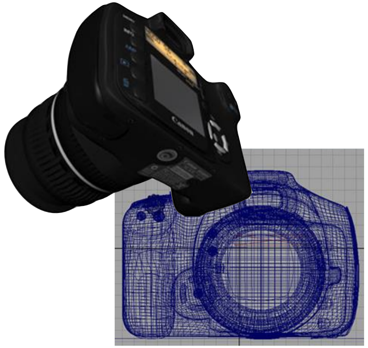
Geometry in Graphics



Applications

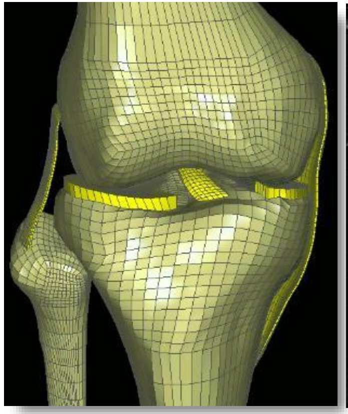
Games/Movies



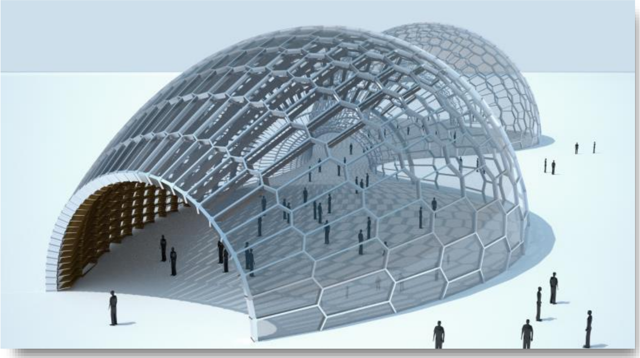
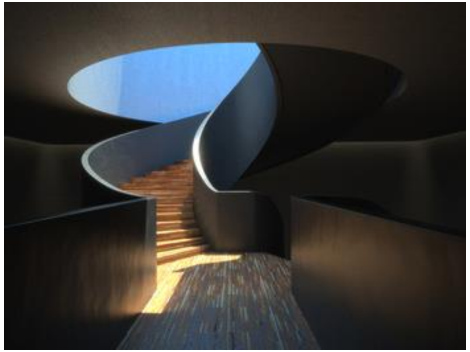


Engineering/Product design

Applications




Medicine/Biology

Architecture

Sources of Geometry

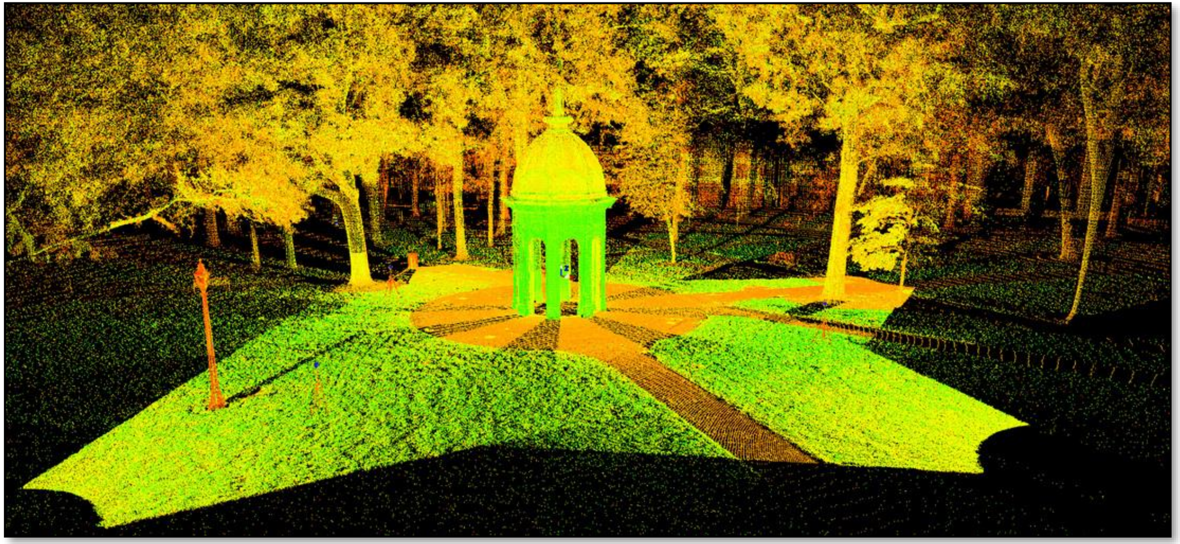
- Acquired real-world objects

3D Scanning

Sources of Geometry

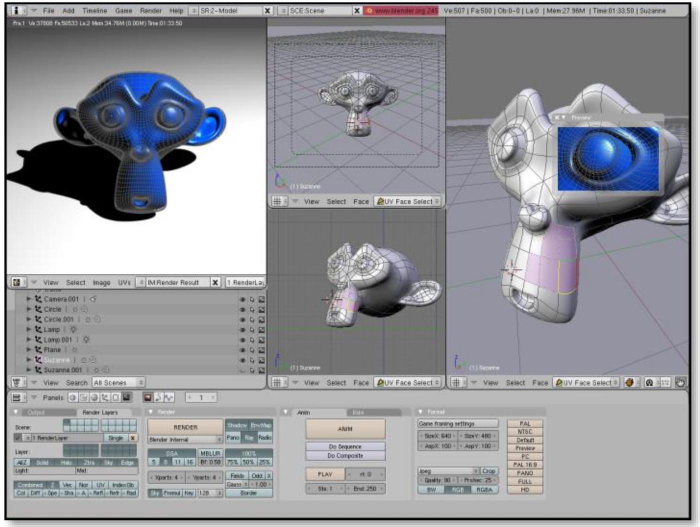
- Acquired real-world objects

Point Clouds



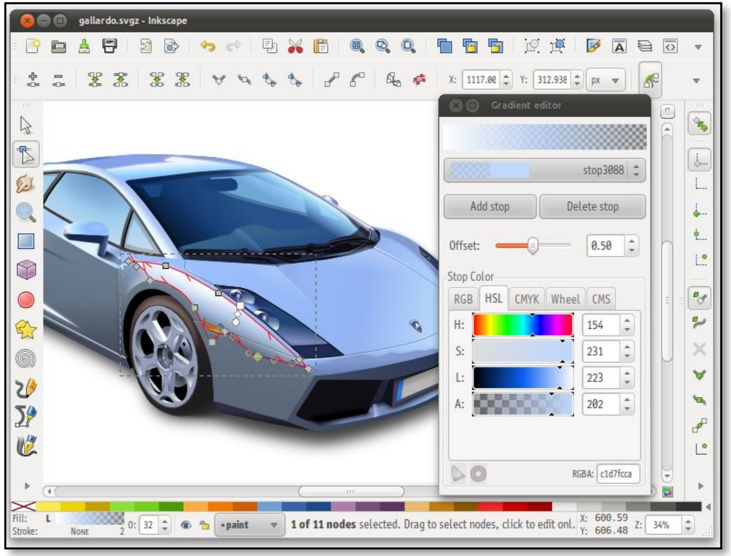
Sources of Geometry

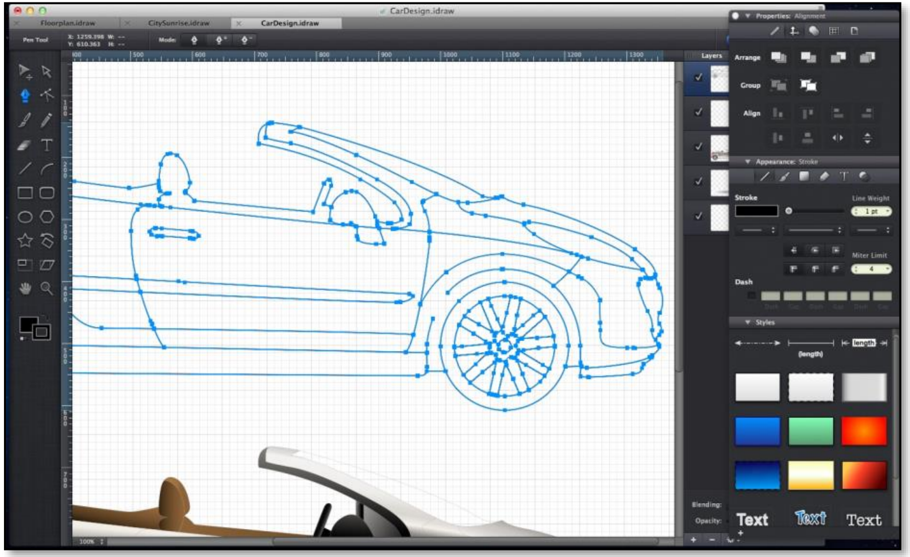
- Digital 3D modeling



Sources of Geometry

- Digital 3D modeling





Sources of Geometry

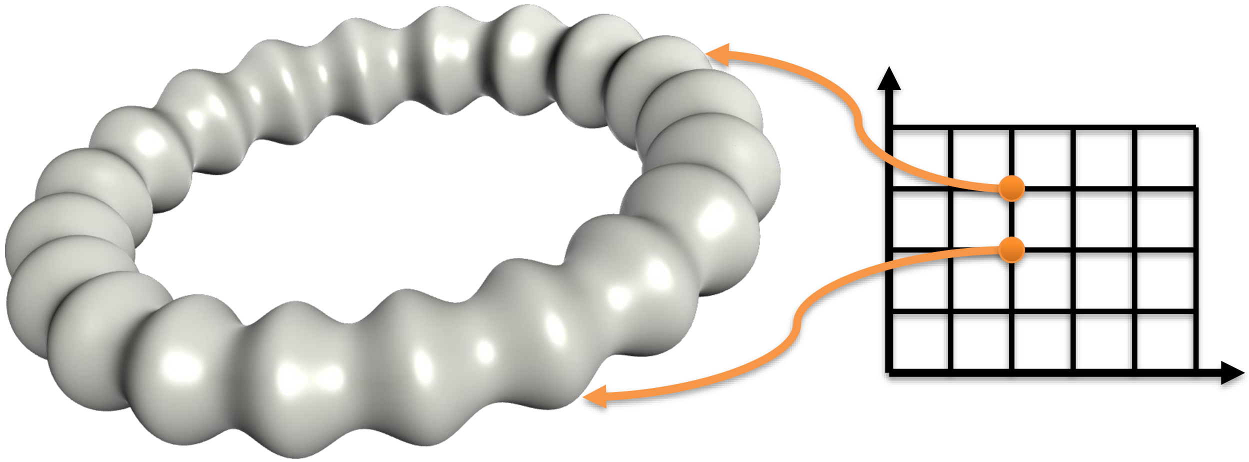
- Procedural Modeling

Geometry Representations

- Considerations
  - Storage
  - Acquisition of shapes
  - Creation of shapes
  - Editing shapes
  - Rendering shapes

Geometry Representations

- Parametric curves & surfaces



$$f : X \to Y, X \subseteq \mathbb{R}^m, Y \subseteq \mathbb{R}^n$$

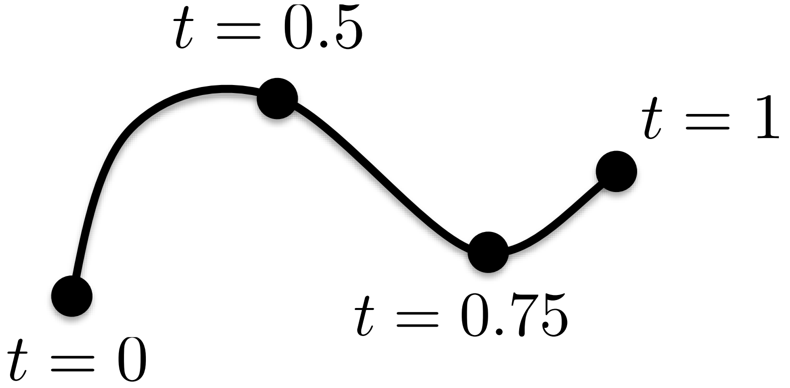
Geometry Representations

- Parametric curves & surfaces

Planar Curves

$$f : X \to Y, X \subseteq \mathbb{R}^m, Y \subseteq \mathbb{R}^n \qquad m = 1, n = 2$$

$$s(t) = (x(t), y(t))$$

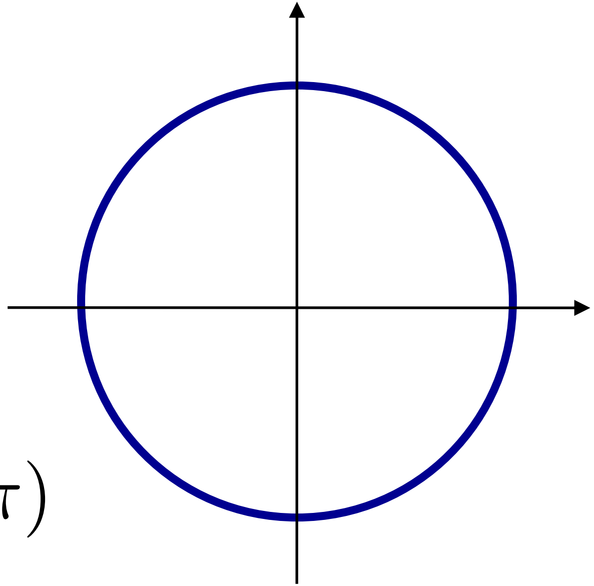


Geometry Representations

- Parametric curves & surfaces

Circle

$$\mathbf{p} : \mathbb{R} \to \mathbb{R}^2$$
$$t \mapsto \mathbf{p}(t) = (x(t), y(t))$$



$$\mathbf{p}(t) = r\,(\cos(t), \sin(t)) \quad t \in [0, 2\pi)$$

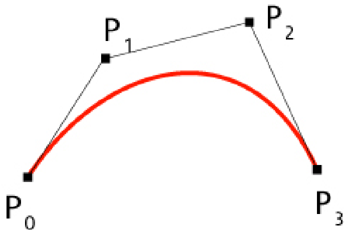
Geometry Representations

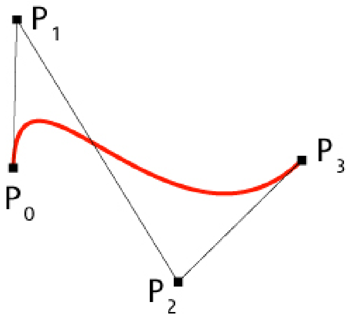
- Parametric curves & surfaces

Bezier Curves

$$s(t) = \sum_{i=0}^{n} \mathbf{p}_i B_i^n(t)$$

$$B_i^n(t) = \binom{n}{i} t^i (1-t)^{n-i}$$





Geometry Representations

- Parametric curves & surfaces

Space Curves in 3D

$$f : X \to Y, X \subseteq \mathbb{R}^m, Y \subseteq \mathbb{R}^n$$

$$m = 1, n = 3$$

$$s(t) = (x(t), y(t), z(t))$$

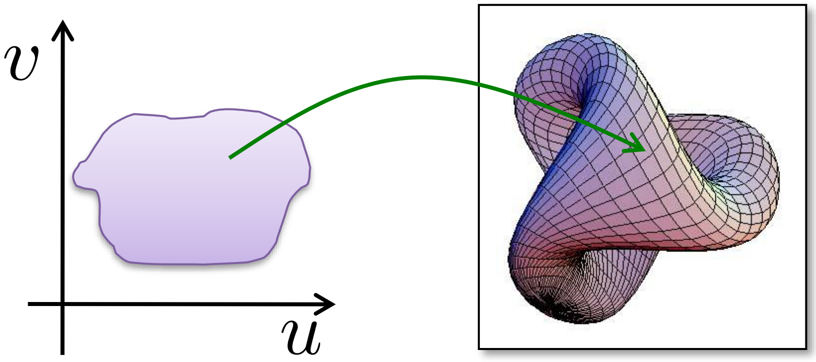
Geometry Representations

- Parametric curves & surfaces

Surfaces

$$f : X \to Y, X \subseteq \mathbb{R}^m, Y \subseteq \mathbb{R}^n \qquad m = 2, n = 3$$

$$s(u, v)$$
$$= (x(u, v), y(u, v), z(u, v))$$



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Geometry Representations

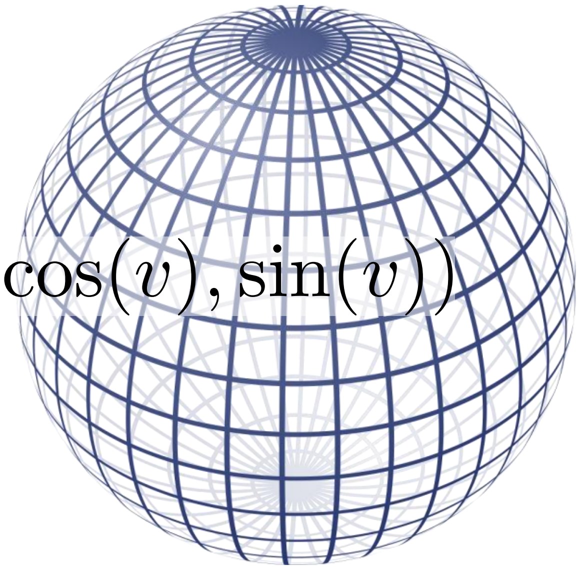
- Parametric curves & surfaces

Sphere

$$s : \mathbb{R}^2 \rightarrow \mathbb{R}^3$$

$$s(u, v) = r\left(\cos(u)\cos(v), \sin(u)\cos(v), \sin(v)\right)$$

$$(u, v) \in [0, 2\pi) \times [-\pi/2, \pi/2]$$

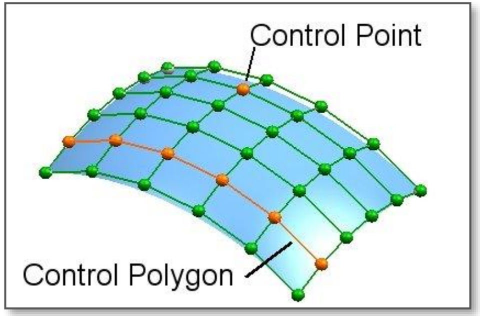


Geometry Representations

- Parametric curves & surfaces

Bezier Surfaces

$$s(u,v) = \sum_{i=0}^{m}\sum_{j=0}^{n} \mathbf{p}_{i,j} B_i^m(u) B_j^n(v)$$



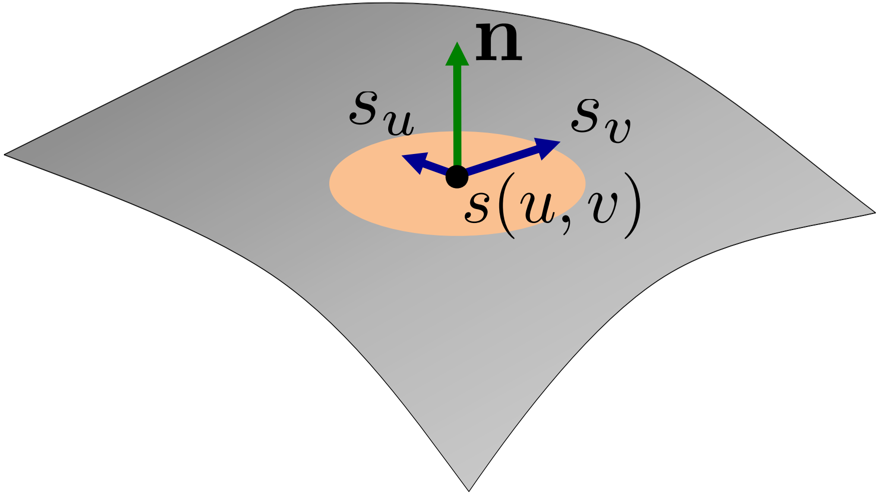
Geometry Representations

- Parametric curves & surfaces

Normal and Tangent plane

$$s_u = \frac{\partial s(u,v)}{\partial u} \qquad s_v = \frac{\partial s(u,v)}{\partial v}$$

$$\mathbf{n} = \frac{s_u \times s_v}{\|s_u \times s_v\|}$$



Geometry Representations

- Parametric curves & surfaces

Volumetric Representations

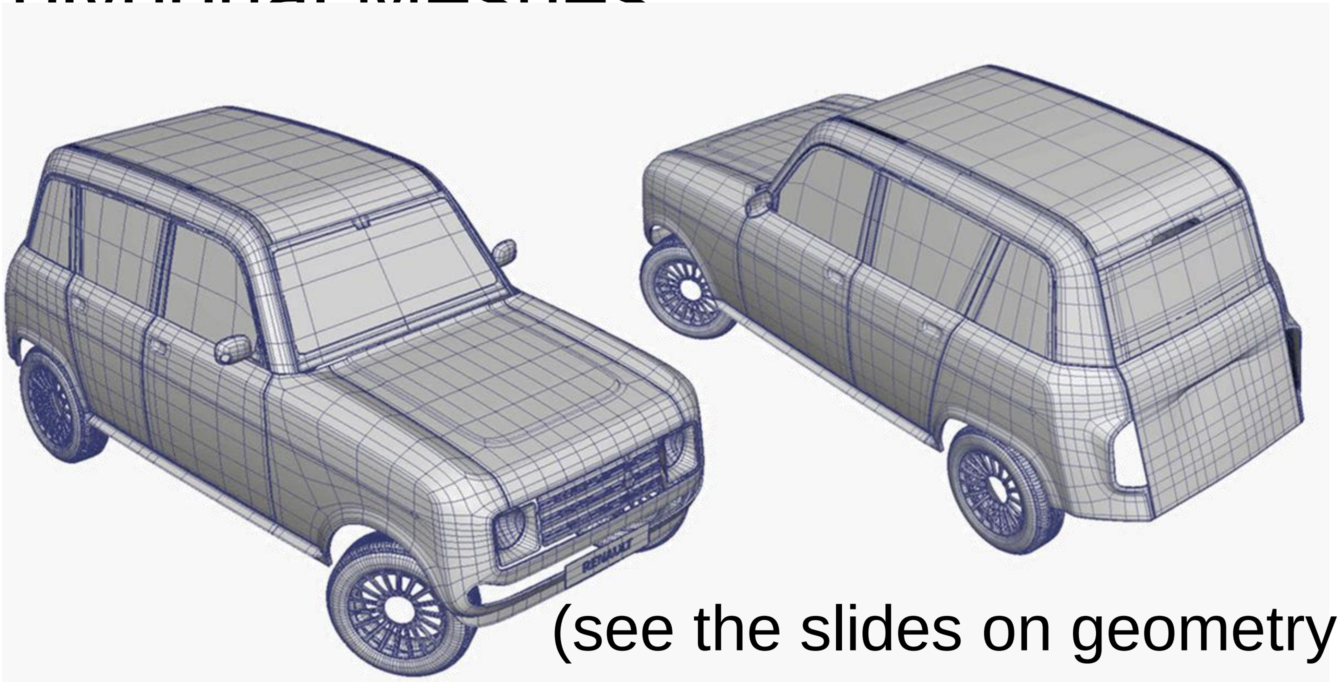
$$f : X \rightarrow Y, X \subseteq \mathbb{R}^m, Y \subseteq \mathbb{R}^n \quad m = 3, n = 1$$

Geometry Representations

- Parametric curves & surfaces
  - \+ Easy to generate points on a curve/surface
  - \+ Easy point-wise differential properties
  - \+ Easy to control by hand
  - − Hard to determine inside/outside
  - − Hard to determine if a point is on a curve/surface
  - − Hard to generate by reverse engineering

Geometry Representations

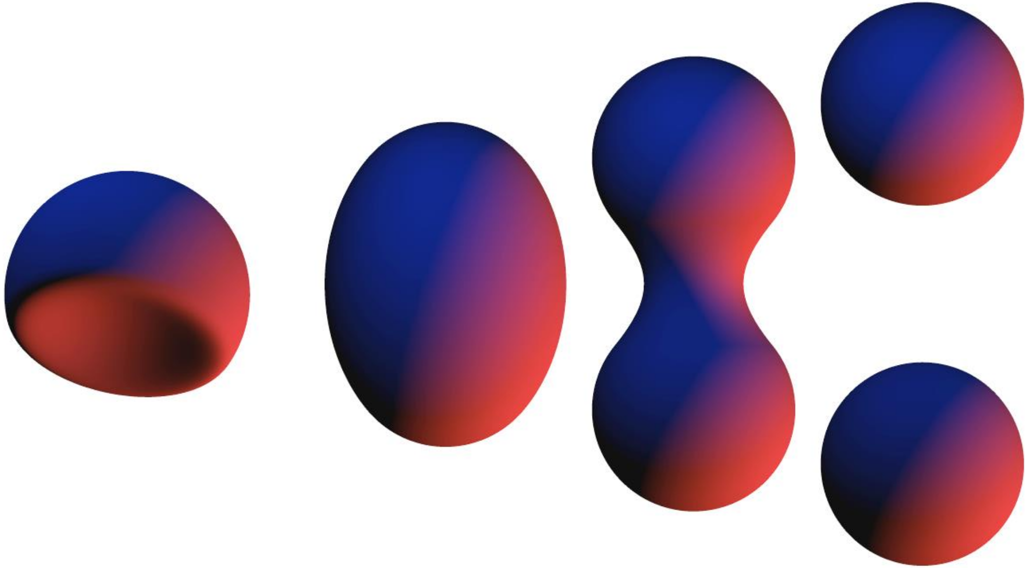
- Polygonal Meshes



(see the slides on geometry & textures)

Geometry Representations

- Implicit surfaces



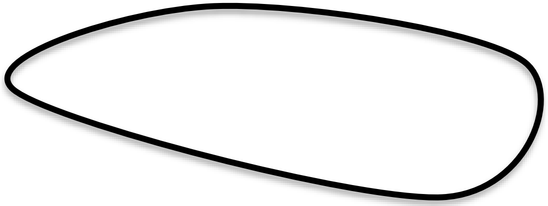
Geometry Representations

- Implicit curves & surfaces

$$f : \mathbb{R}^m \rightarrow \mathbb{R}$$

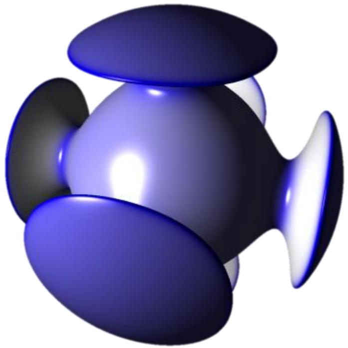
Planar Curves

$$S = \{x \in \mathbb{R}^2 | f(x) = 0\}$$



Surfaces in 3D

$$S = \{x \in \mathbb{R}^3 | f(x) = 0\}$$



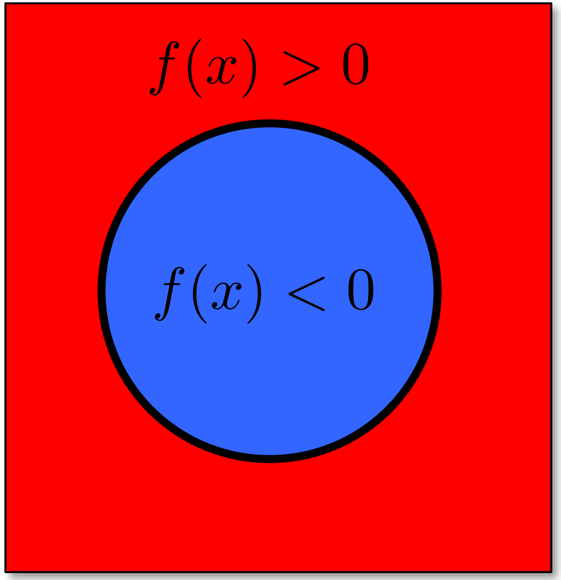
Geometry Representations

- Implicit curves & surfaces

$$\{x \in \mathbb{R}^m | f(x) > 0\}$$ Outside

$$\{x \in \mathbb{R}^m | f(x) = 0\}$$ Curve/Surface

$$\{x \in \mathbb{R}^m | f(x) < 0\}$$ Inside

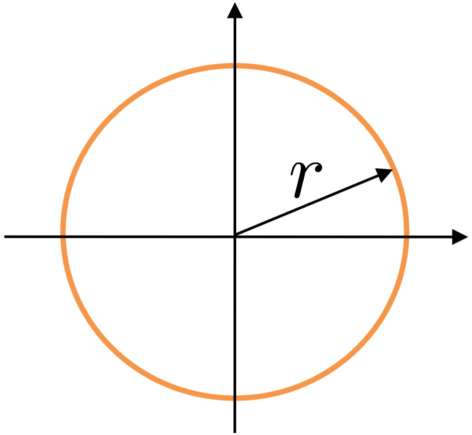


Geometry Representations

- Implicit curves & surfaces

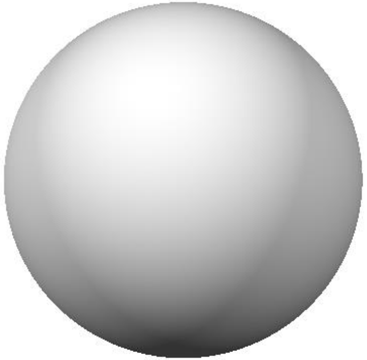
Circle

$$f(x,y) = x^2 + y^2 - r^2$$



Sphere

$$f(x,y,z) = x^2 + y^2 + z^2 - r^2$$



Geometry Representations

- Implicit curves & surfaces

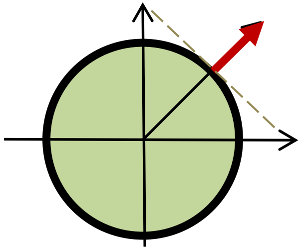
Surface Normal

$$\nabla f(x,y,z) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)^T$$

Circle

$$f(x,y,z) = x^2 + y^2 + z^2 - r^2$$

$$\nabla f(x,y,z) = (2x, 2y, 2z)^T$$

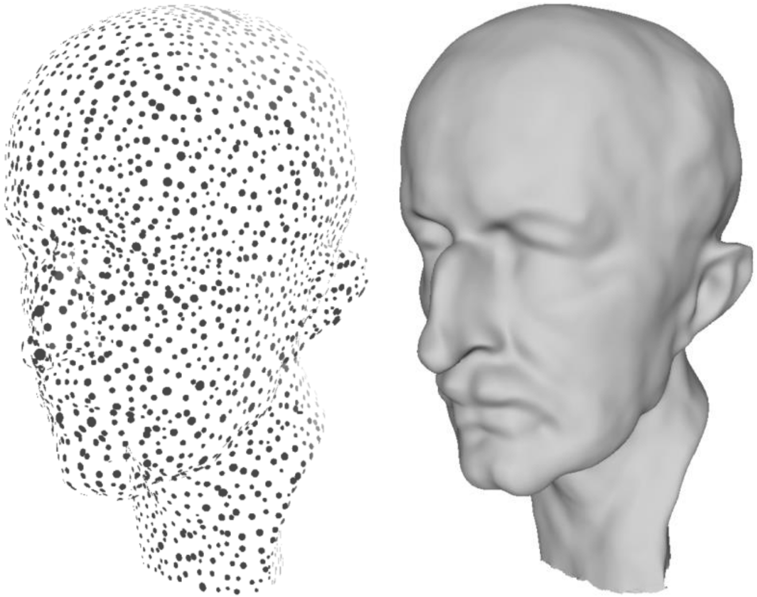


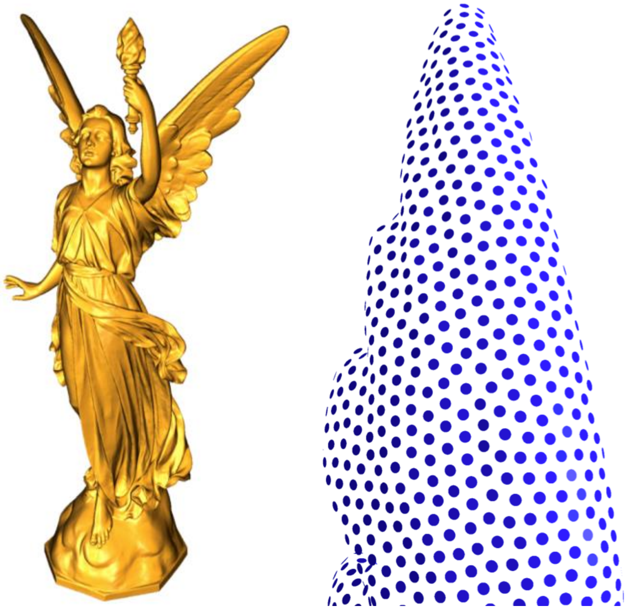
Geometry Representations

- Implicit curves & surfaces
  - + Easy to determine inside/outside
  - + Easy to determine if a point is on a curve/surface
  - + Easy to combine
  - − Hard to generate points on a curve/surface
  - − Limited set of surfaces
  - − Does not lend itself to (real-time) rendering

Geometry Representations

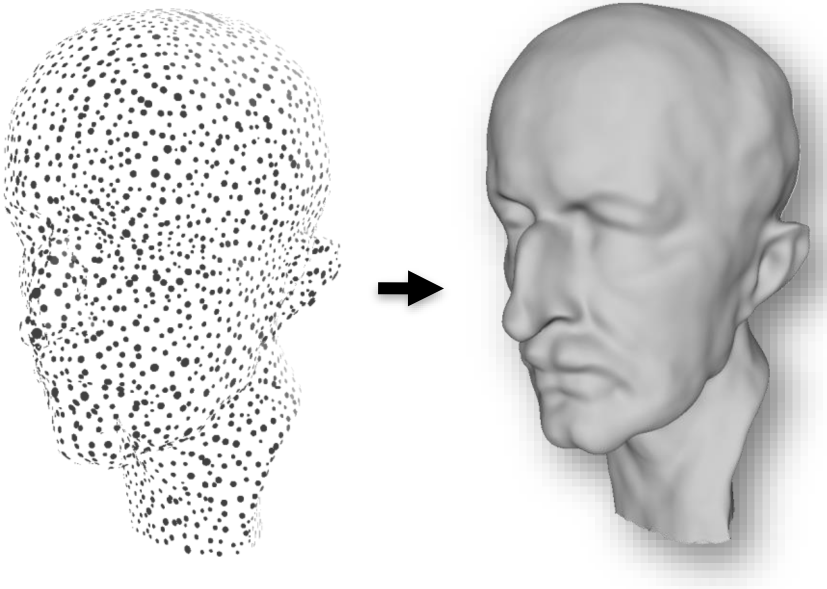
- Point Set Surfaces





Geometry Representations

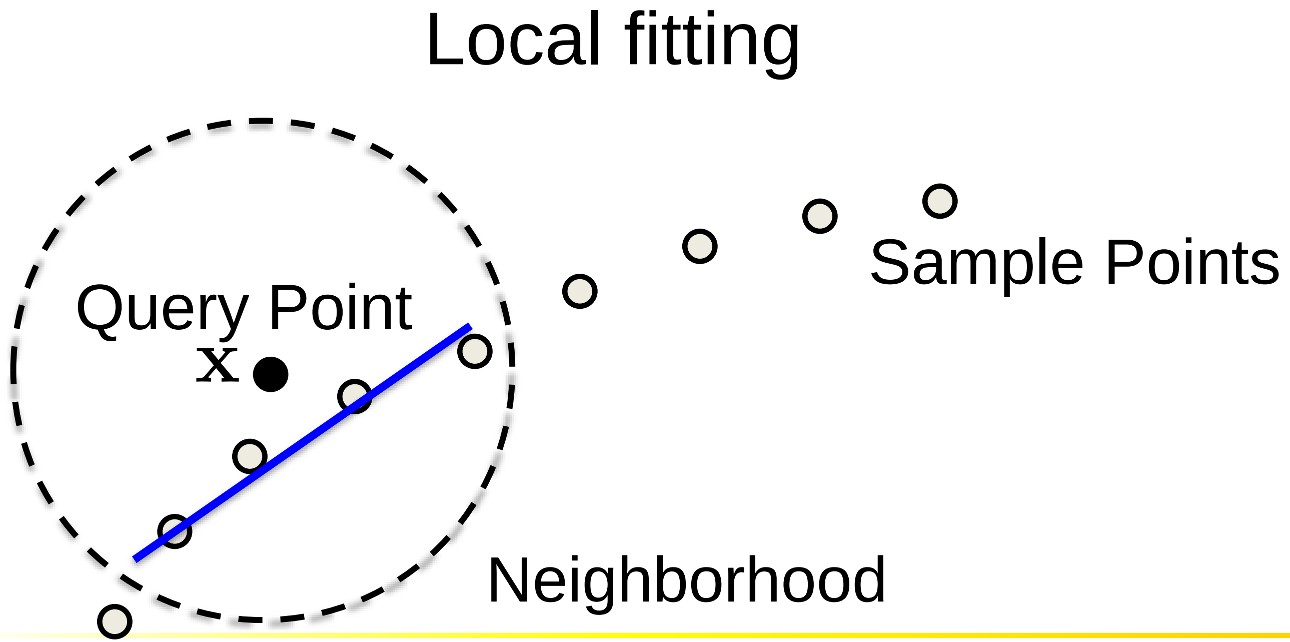
- Point Set Surfaces



Only point-wise attributes
Approximation methods
Smooth surfaces
Works on acquired data

Geometry Representations

- Point Set Surfaces

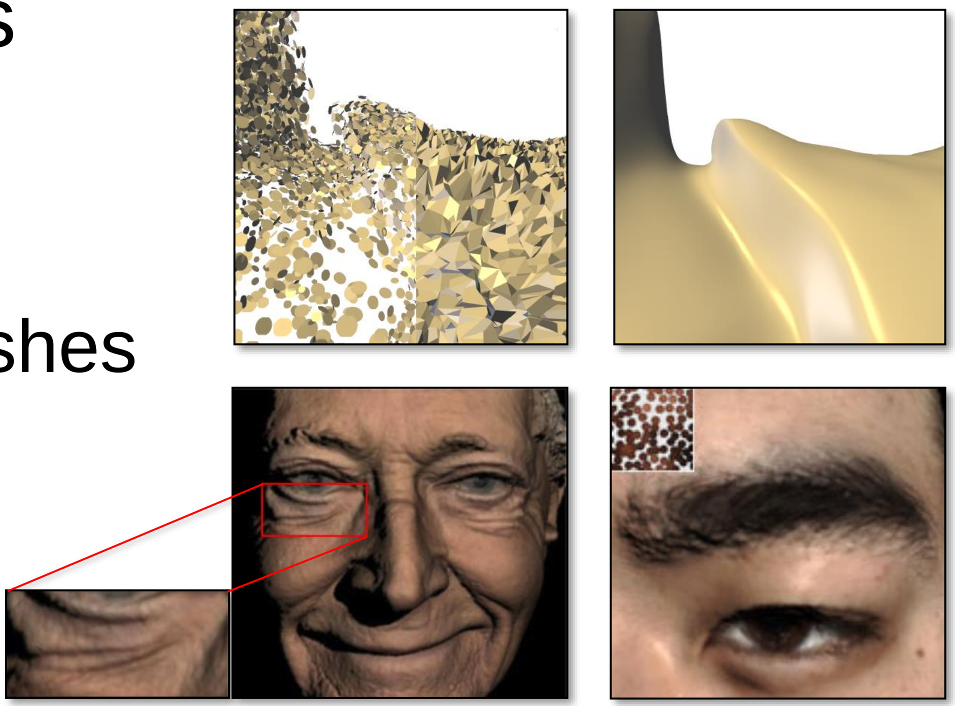


Geometry Representations

- Point Set Surfaces
  - Implicit representation & fast projection

Geometry Representations

- Point Set Surfaces
  - Robust to noise
  - Direct rendering
  - Conversion to meshes



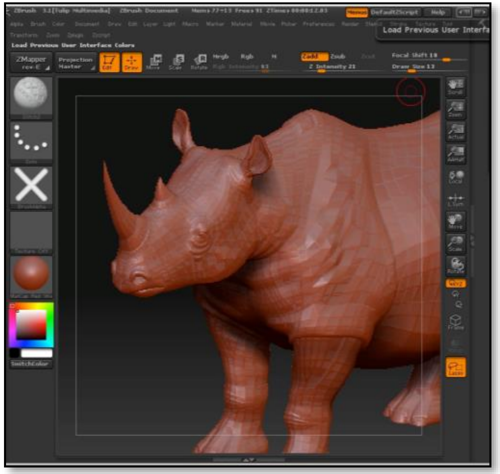
Geometry Representations

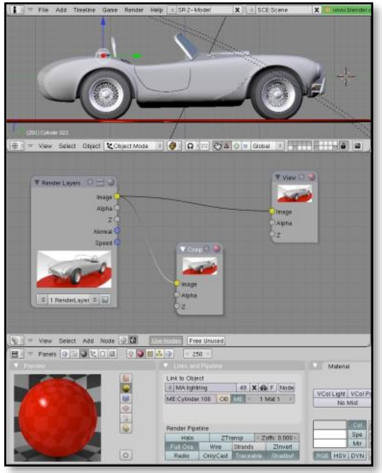
- Point Set Surfaces
  - \+ Easy to determine inside/outside
  - \+ Easy to determine if a point is on the curve/surface
  - \+ Easy to generate points on the curve/surface
  - \+ Suitable for reconstruction from general data
  - \+ Direct real-time rendering
  - − Not efficient to use in some modeling tasks

Sources of Geometry

Acquisition from the real world

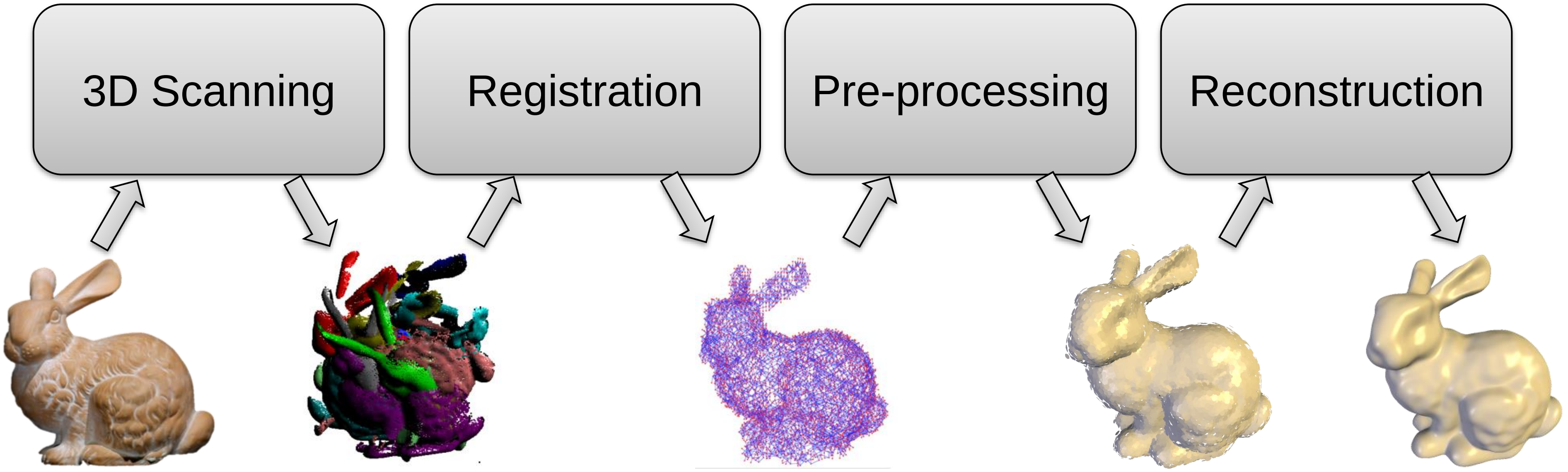
Modeling applications





Shape Acquisition

- Digitalizing real world objects



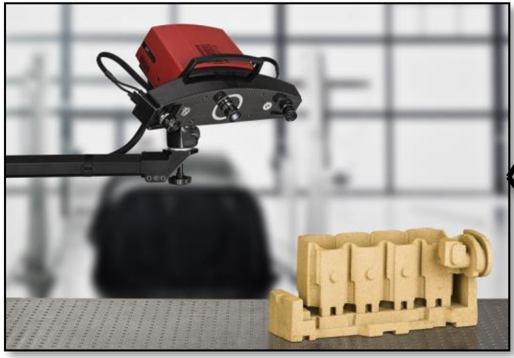
Shape Acquisition

- 3D Scanning

Touch Probes

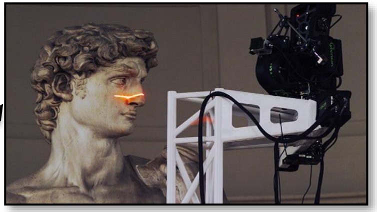
+ Precise
- Small objects

Optical Scanning



+ Fast
- Glossy objects

Active



Passive

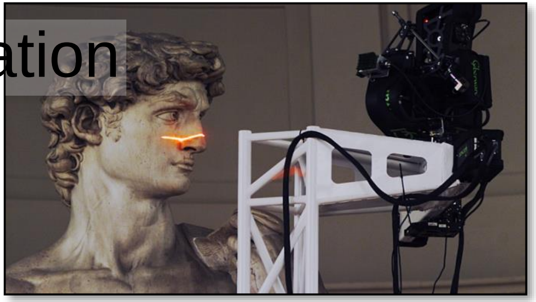
Shape Acquisition

- Optical Scanning – Active Systems

LIDAR

Measures the time it takes the laser beam to hit the object and come back

Triangulation Laser

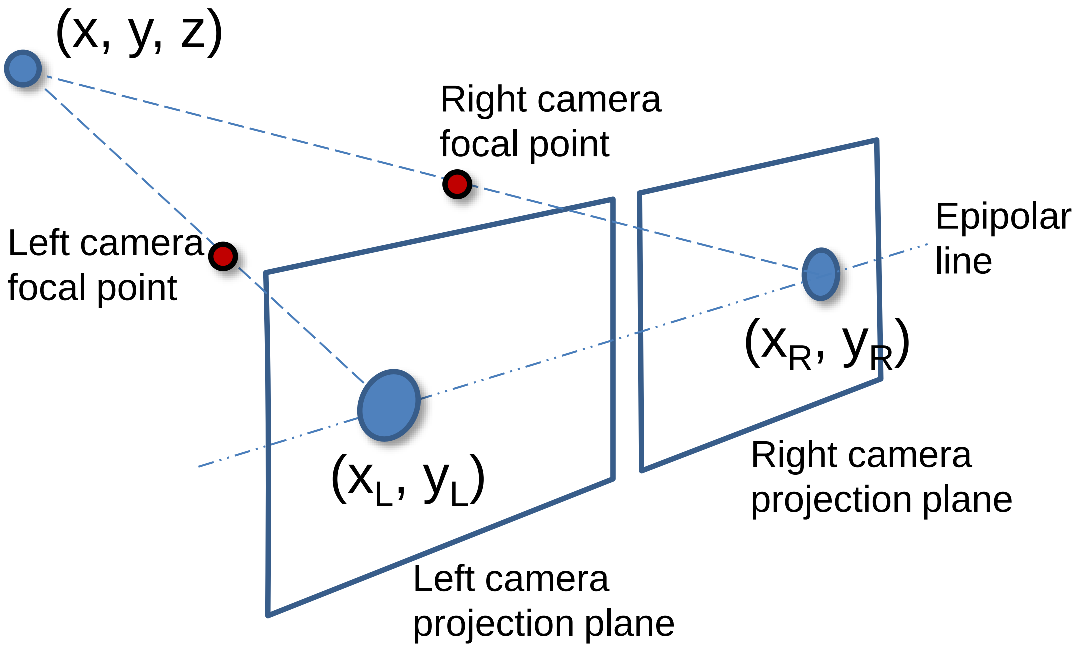


Projected laser beam is photographed, giving the distance of the pattern

Shape Acquisition

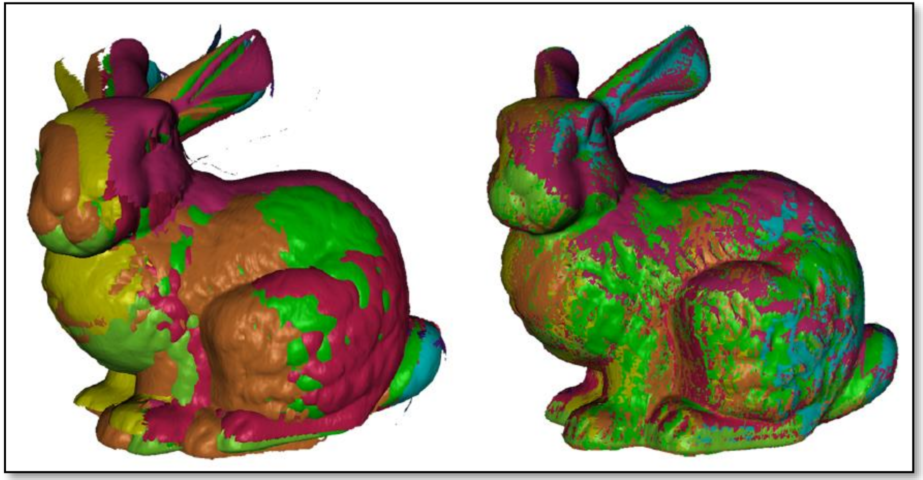
- Optical Scanning – Passive Systems

Multi-view Stereo



Shape Acquisition

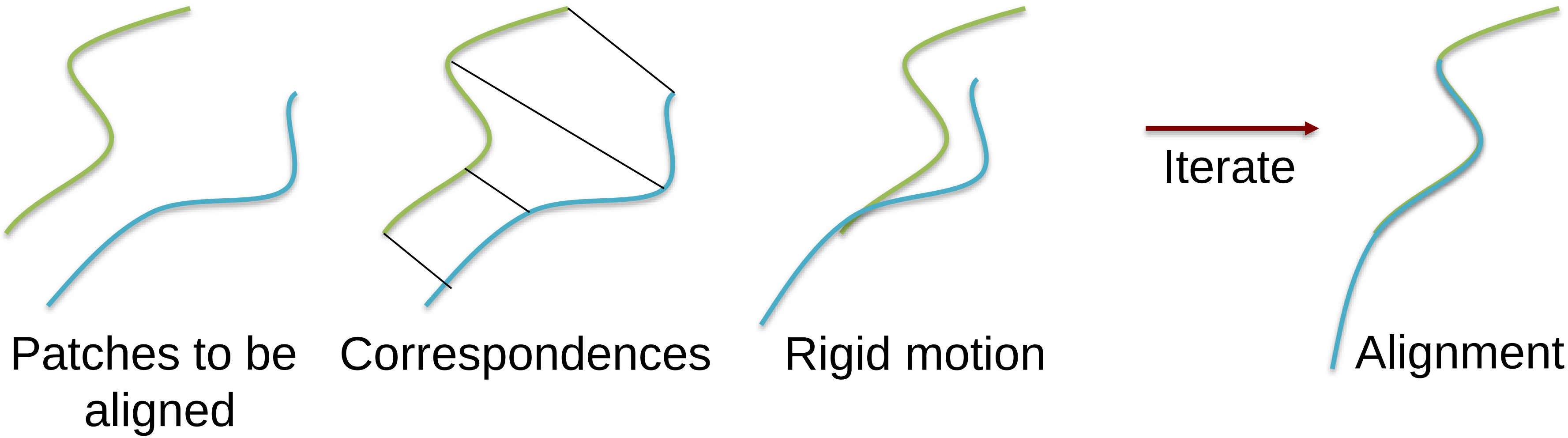
- Registration
  - Bringing scans into a common coordinate frame



Shape Acquisition

- Registration

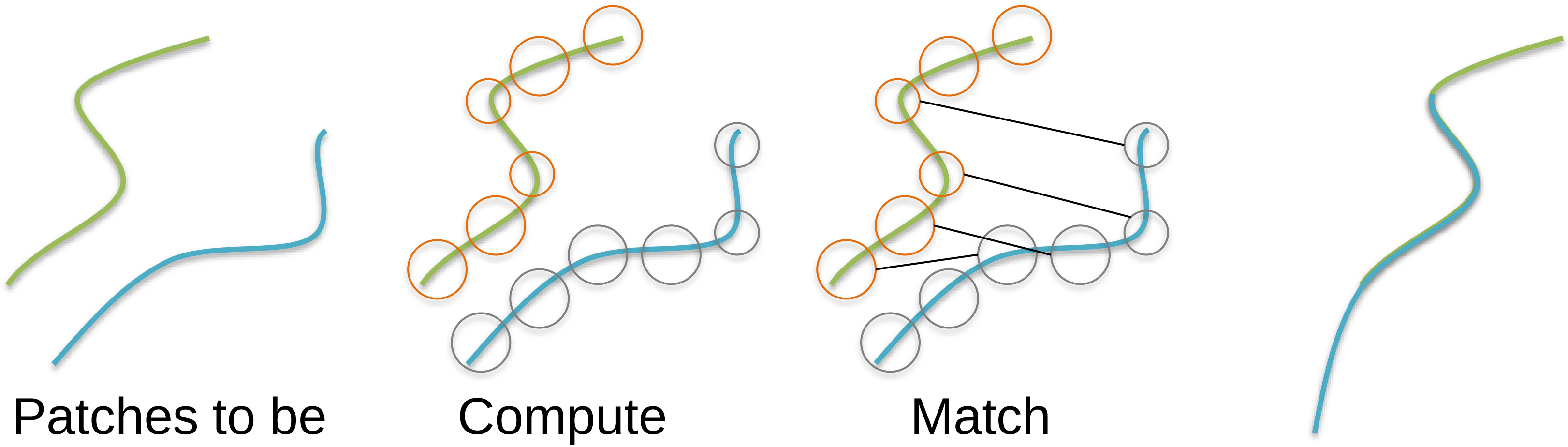
Iterative Closest Point Algorithms



Shape Acquisition

- Registration

Feature-based Methods



Patches to be aligned

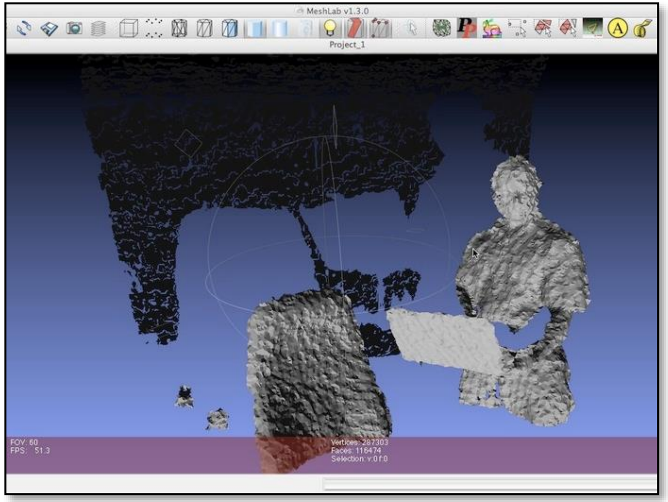
Compute descriptors

Match descriptors

Alignment

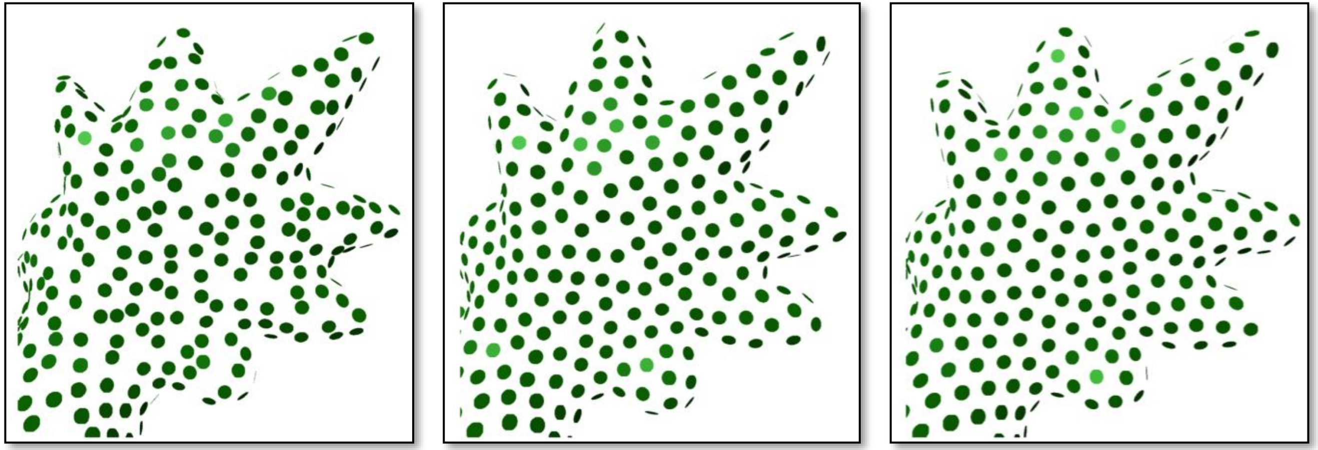
Shape Acquisition

- Pre-processing
  - Cleaning, repairing, resampling



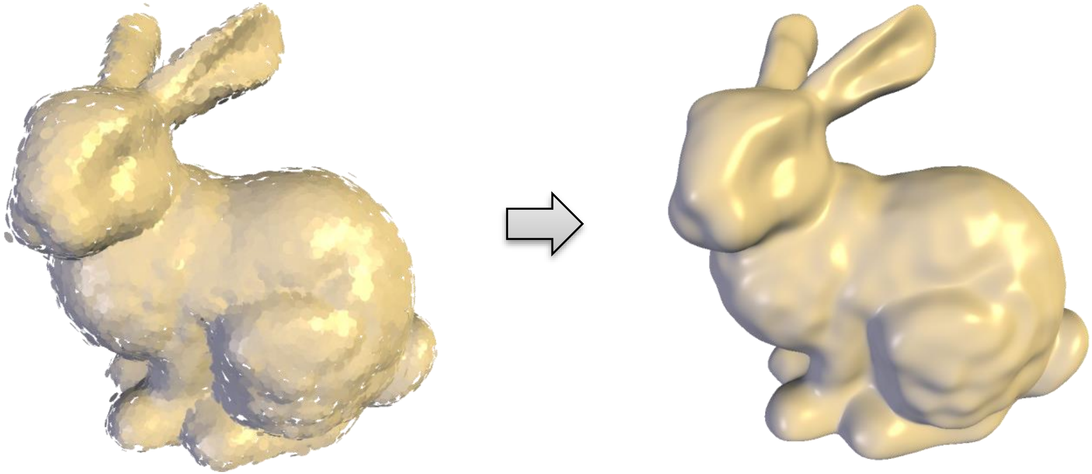
Shape Acquisition

- Pre-processing
  - Sampling for accurate reconstructions



Shape Acquisition

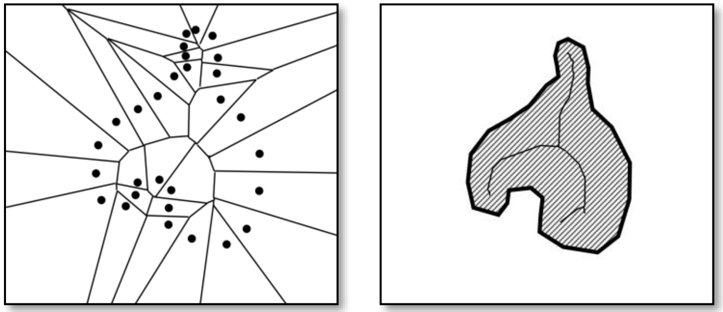
- Reconstruction
  - Mathematical representation for a shape



Shape Acquisition

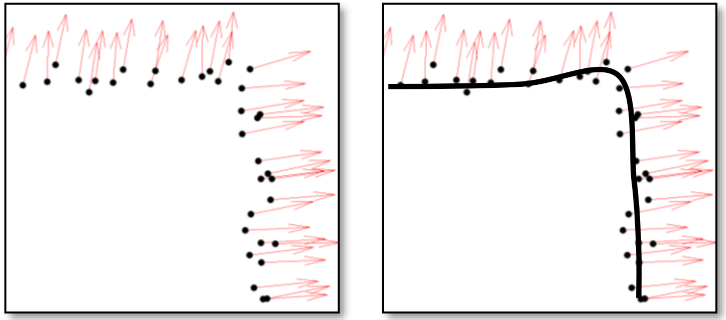
- Reconstruction

Connect-the-points Methods



+ Theoretical error bounds
– Expensive
– Not robust to noise

Approximation-based Methods



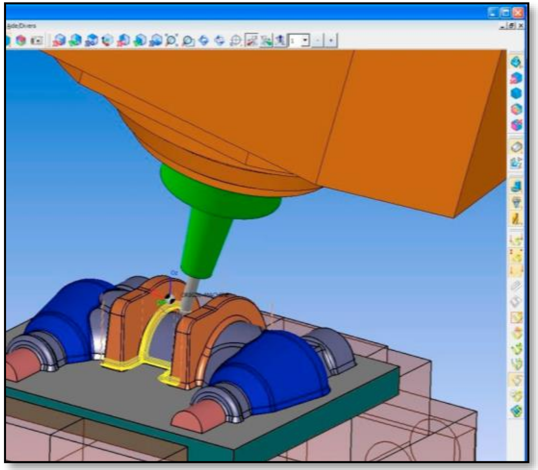
+ Efficient to compute
+ Robust to noise
– No theoretical error bounds

Modeling Tools

- Modeling tools

Sculpting

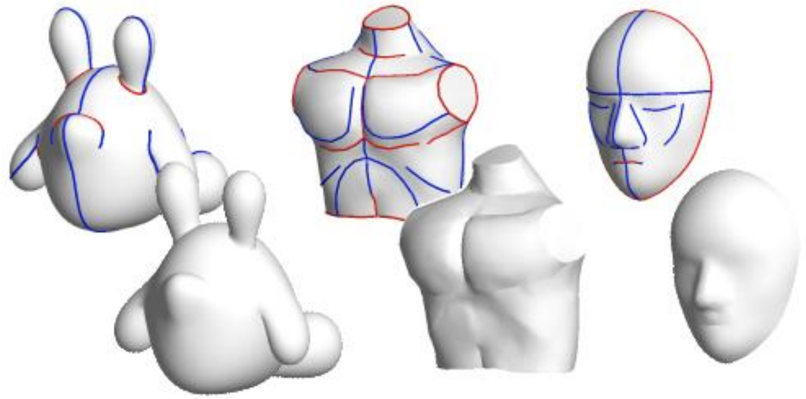
CAD/CAM

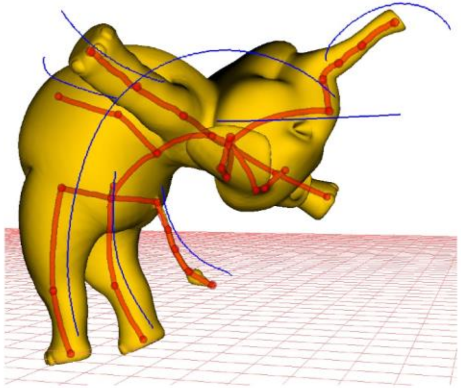


Procedural

Editing Geometry

- Interactive & sketch-based interfaces

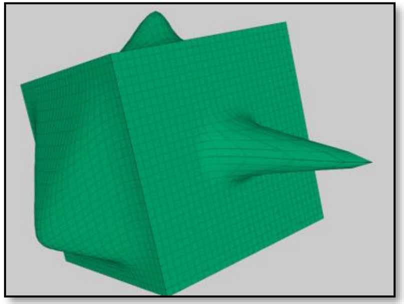




Editing Geometry

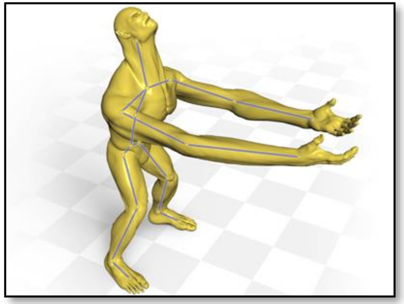
- Deformations

Free-form

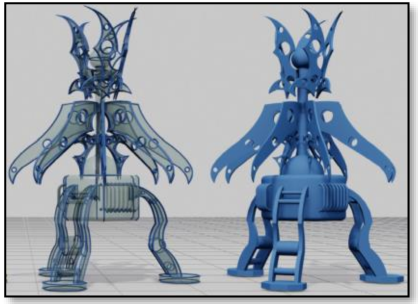


Elastic

Skeletal



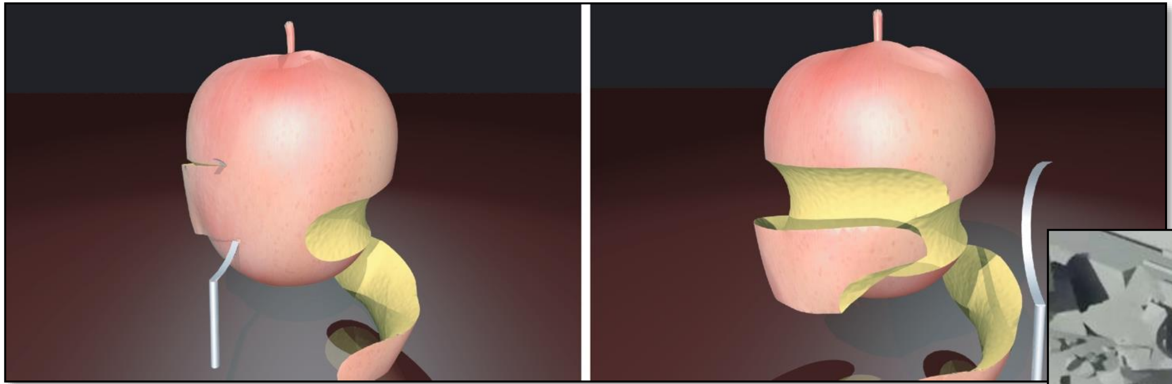
Structure-aware

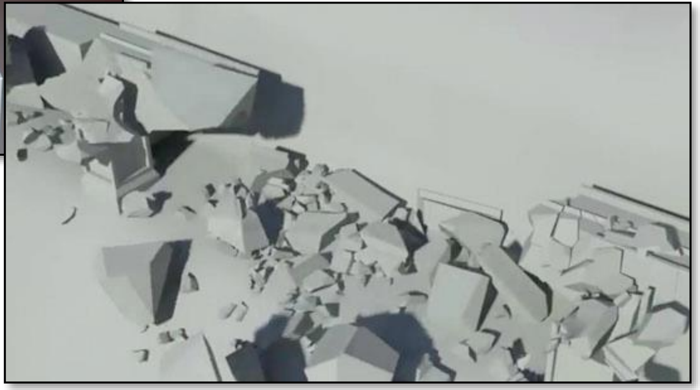


More structure

Editing Geometry

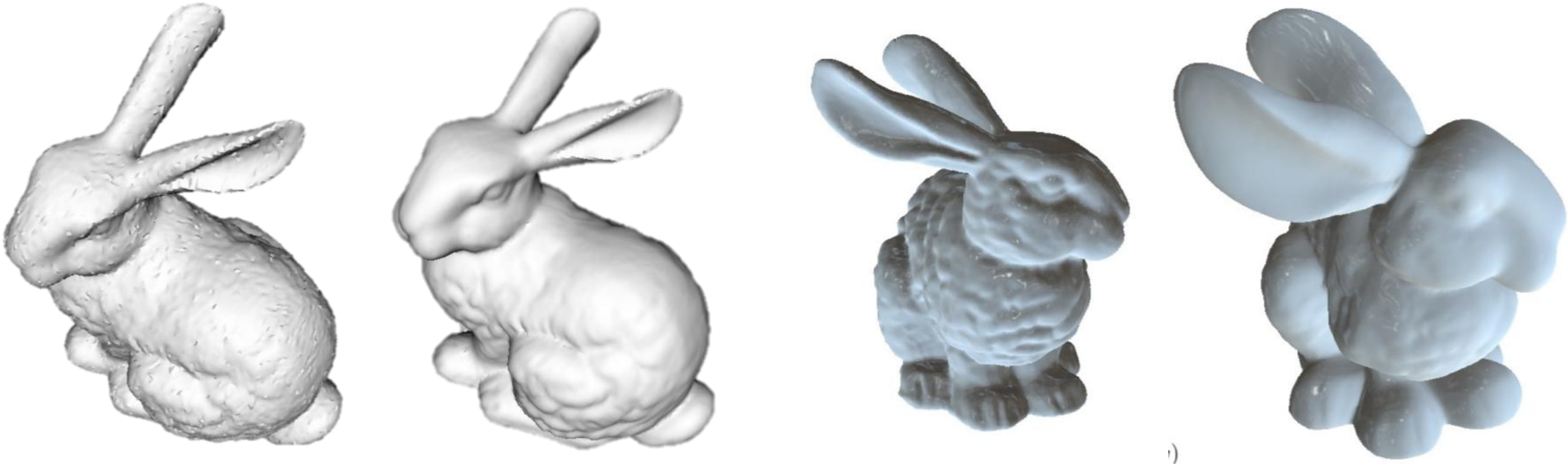
- Cutting & fracturing





Editing Geometry

- Smoothing & filtering



Editing Geometry

- Filtering with the Laplace-Beltrami operator
  - Laplace-Beltrami operator analogue of Laplace
  - Operates on functions on a manifold
  - Each manifold *M* has a different LB operator
  - Described by the eigenvalues/eigenfunctions

$$\Delta_M \phi(x) = \lambda \phi(x)$$

Laplace-Beltrami Eigenfunctions Eigenvalues

Editing Geometry

- Filtering with the Laplace-Beltrami operator
  - Fourier basis are eigenfunctions of Laplace

$$\Delta e^{i2\pi wx} = \frac{\partial^2 e^{i2\pi wx}}{\partial^2 x} = -(2\pi w)^2 e^{i2\pi wx}$$

  - For manifolds: Laplace-Beltrami

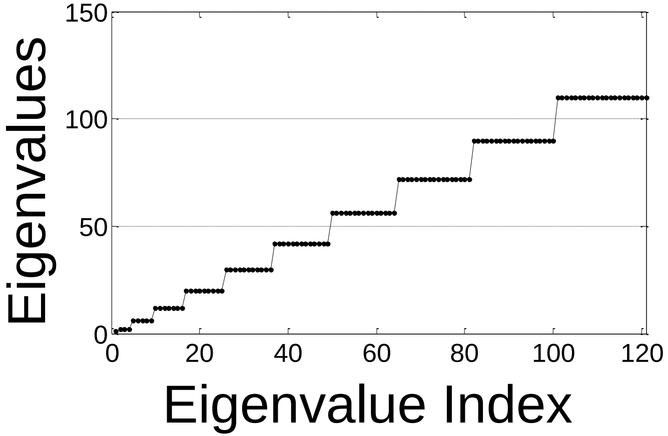
$$\Delta_M \phi(x) = \lambda \phi(x)$$

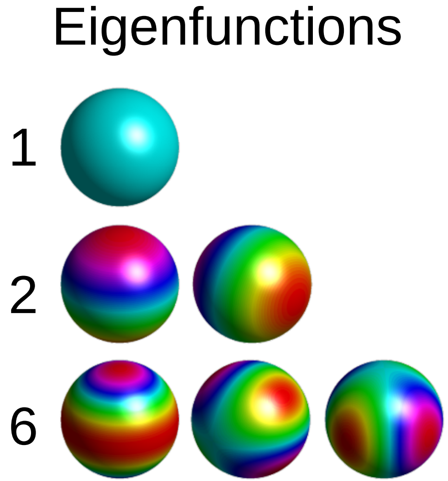
Laplace-Beltrami Eigenfunctions Eigenvalues

Editing Geometry

- Filtering with the Laplace-Beltrami operator

Sphere

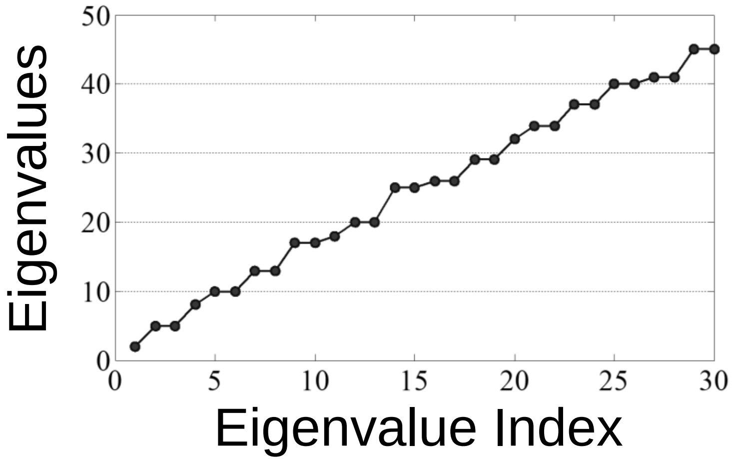




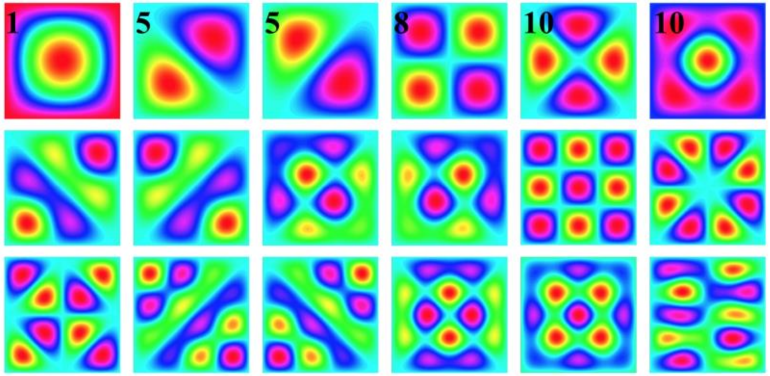
Editing Geometry

- Filtering with the Laplace-Beltrami operator

Square



Eigenfunctions

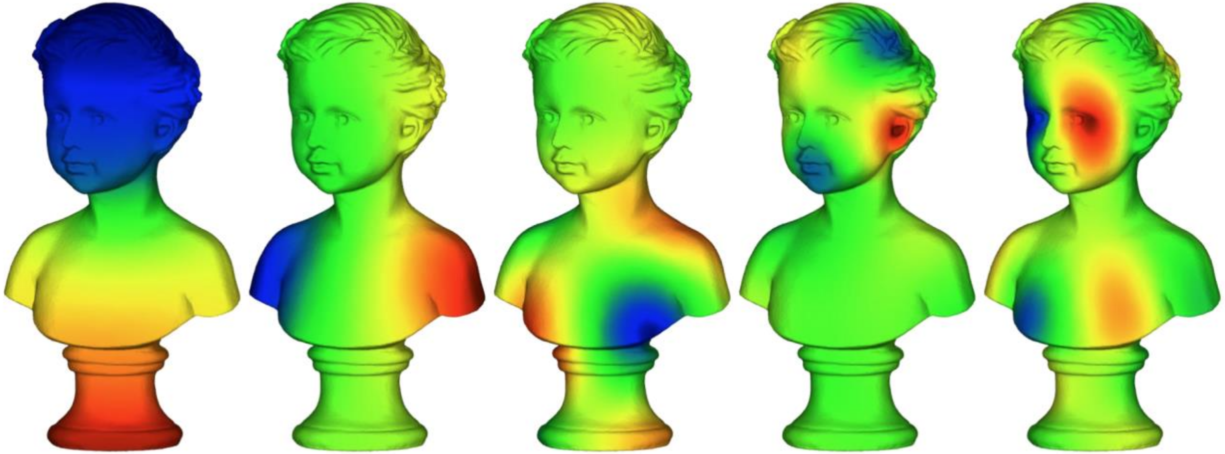


Editing Geometry

- Filtering with the Laplace-Beltrami operator

  Surfaces

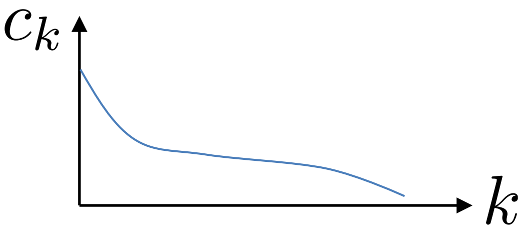
Eigenfunctions



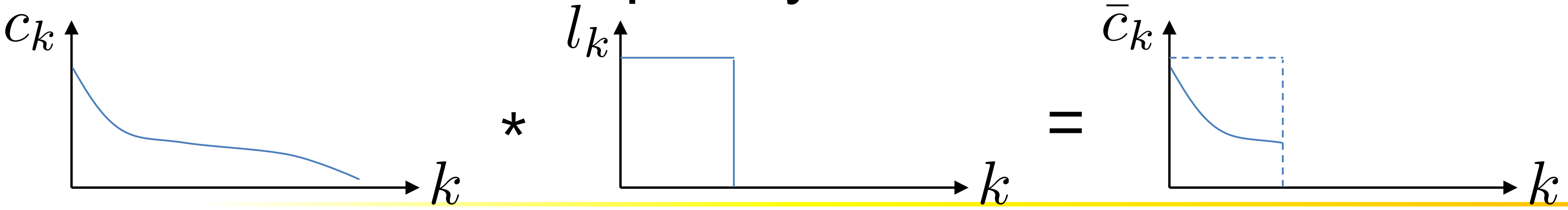
Editing Geometry

- Filtering with the Laplace-Beltrami operator
  - Analogue of Fourier transform on manifolds

$$c_k = \int f(x)\phi_k(x)$$



  - Filter in the “frequency” domain



Editing Geometry

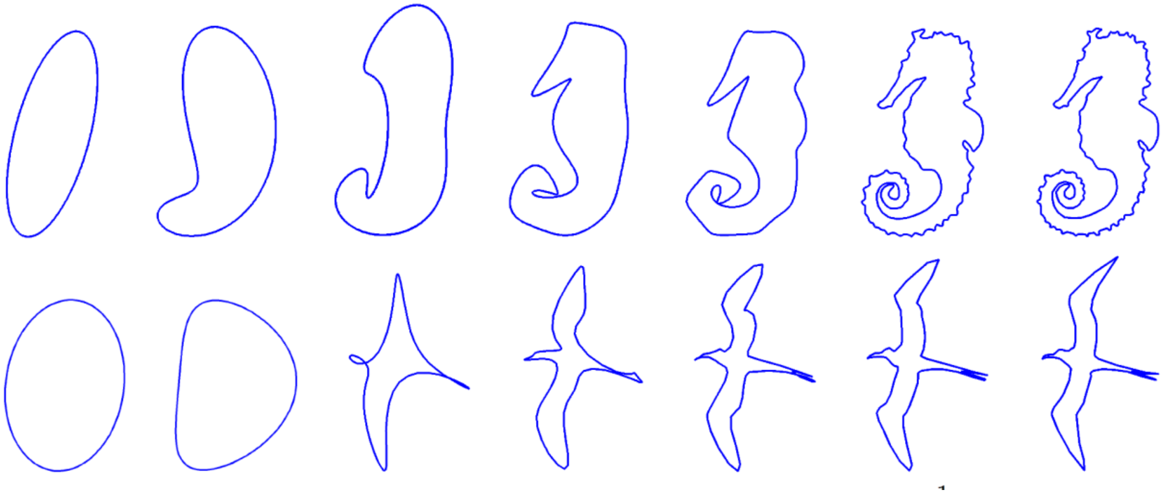
- Filtering with the Laplace-Beltrami operator
  - Allows multi-scale analysis on manifolds
  - Coordinate functions

$$f_1(x \in M) = \mathbf{x}_x \quad f_2(x \in M) = \mathbf{x}_y \quad f_3(x \in M) = \mathbf{x}_z$$

  - Filter coordinate functions (coordinates of the points) on the manifold

Editing Geometry

- Filtering with the Laplace-Beltrami operator
  - Filter coordinates of the points on the manifold

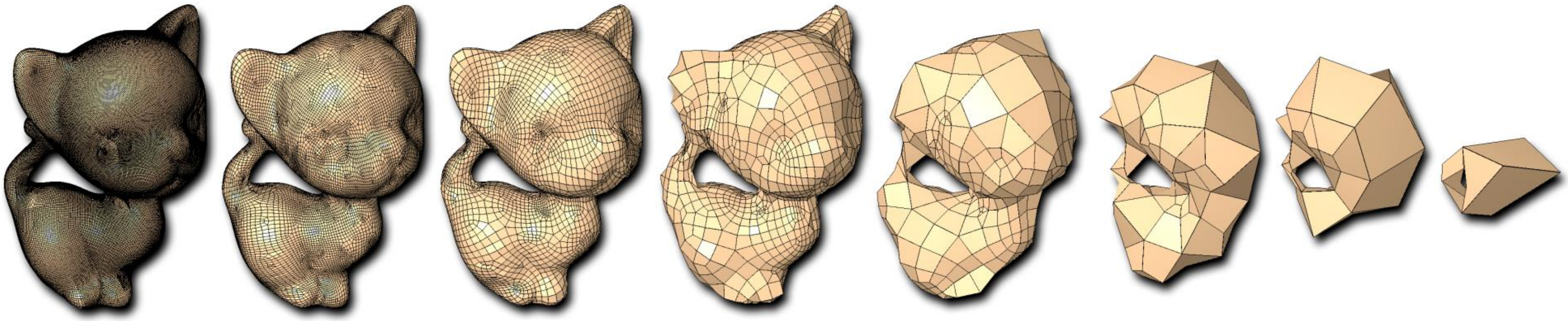


Editing Geometry

- Filtering with the Laplace-Beltrami operator
  - Filter coordinates of the points on the manifold

Editing Geometry

- Compression & Simplification



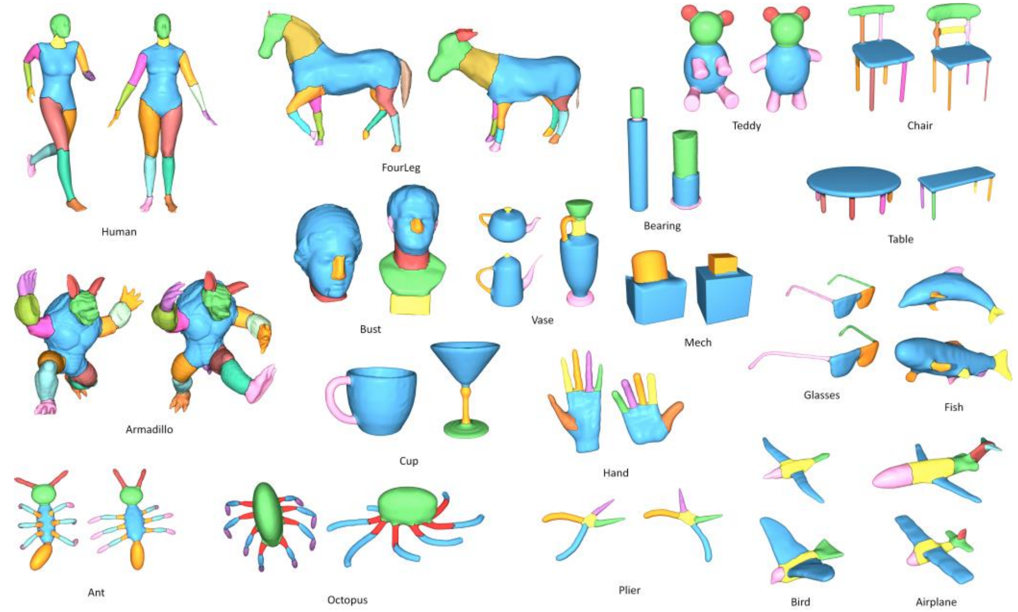
Analysis of Geometry

- Differential Properties

Mean Absolute Curvature

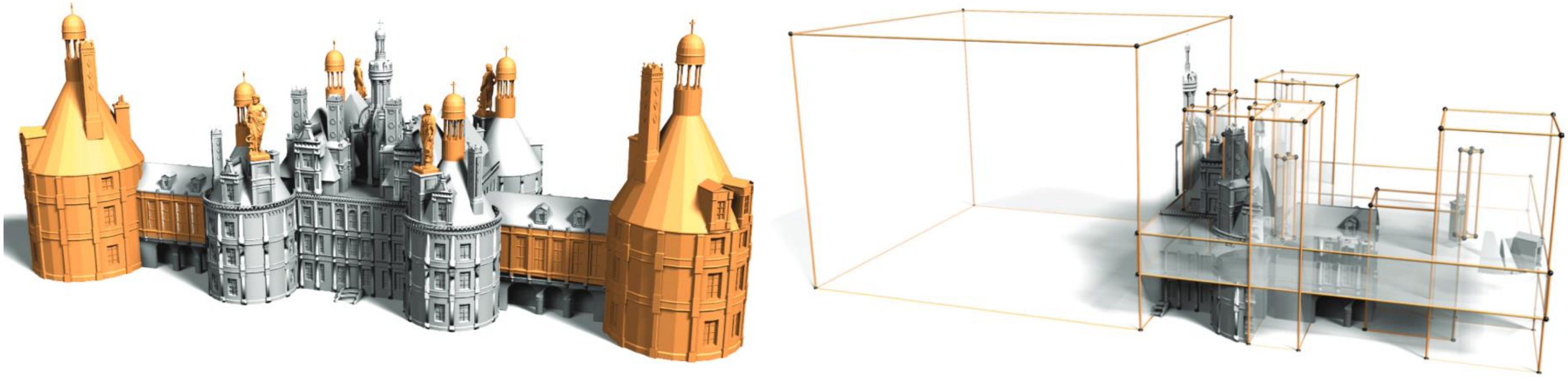
Analysis of Geometry

- Segmentation



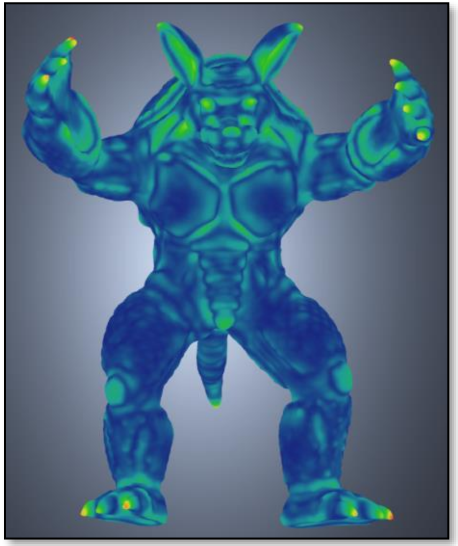
Analysis of Geometry

- Symmetry and structure detection



Analysis of Geometry

- Saliency



Analysis of Geometry

- Feature Extraction