3D Vision

Marc Pollefeys, Daniel Barath

Spring 2024

Schedule

Feb 19	Introduction
Feb 26	Geometry, Camera Model, Calibration
Mar 4	Guest lecture + Features, Tracking / Matching
Mar 11	**Project Proposals by Students**
Mar 18	3DV conference
Mar 25	Structure from Motion (SfM) + papers
Apr 1	Easter break
Apr 8	Dense Correspondence (stereo / optical flow) + papers
Apr 15	**Bundle Adjustment & SLAM + papers**
Apr 22	**Student Midterm Presentations**
Apr 29	Multi-View Stereo & Volumetric Modeling + papers
May 6	3D Modeling with Depth Sensors + papers
May 13	Guest lecture + papers
May 20	Holiday

3D Vision – Class 6

Bundle Adjustment and SLAM

- [Triggs, McLauchlan, Hartley, Fitzgibbon, Bundle Adjustment – A Modern Synthesis, Int. Workshop on Vision Algorithms, 1999]
- [Montemerio, Thrun, Koller, Wegbreit, FastSLAM: A Factored Solution to the Simultaneous Localization and Mapping Problem, AAAI 2002]
- Section 2.5 from [Lee, Visual Mapping and Pose Estimation for Self-Driving Cars, PhD Thesis, ETH Zurich, 2014]

Slides : Gim Hee Lee

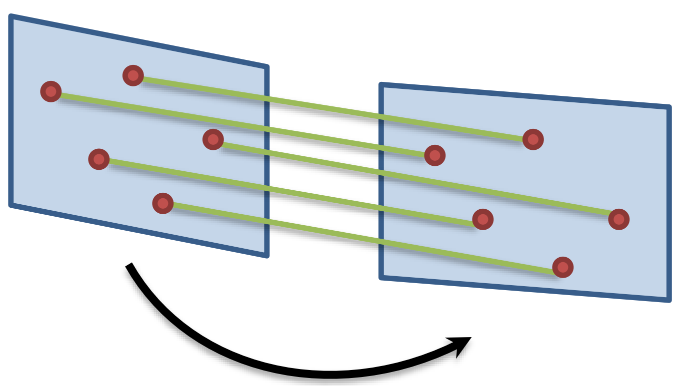
ETH

Lecture Overview

- Bundle Adjustment in Structure-from-Motion
- Simultaneous Localization & Mapping (SLAM)

Recap: Structure-From-Motion

- Two views initialization:
  - 5-Point algorithm (Minimal Solver)
  - 8-Point linear algorithm
  - 7-Point algorithm

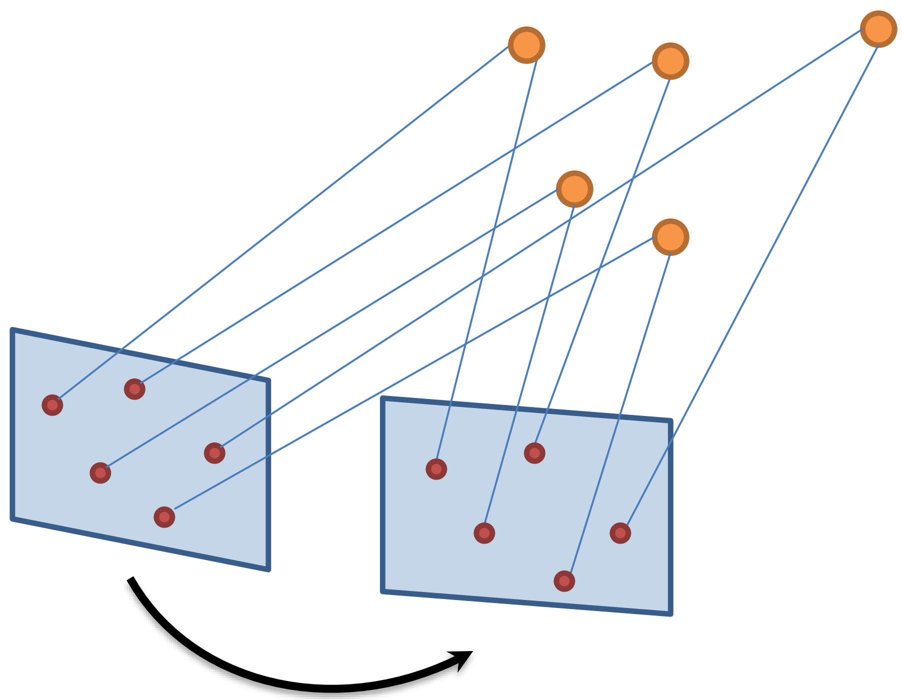


E → (R,t)

ETH

Recap: Structure-From-Motion

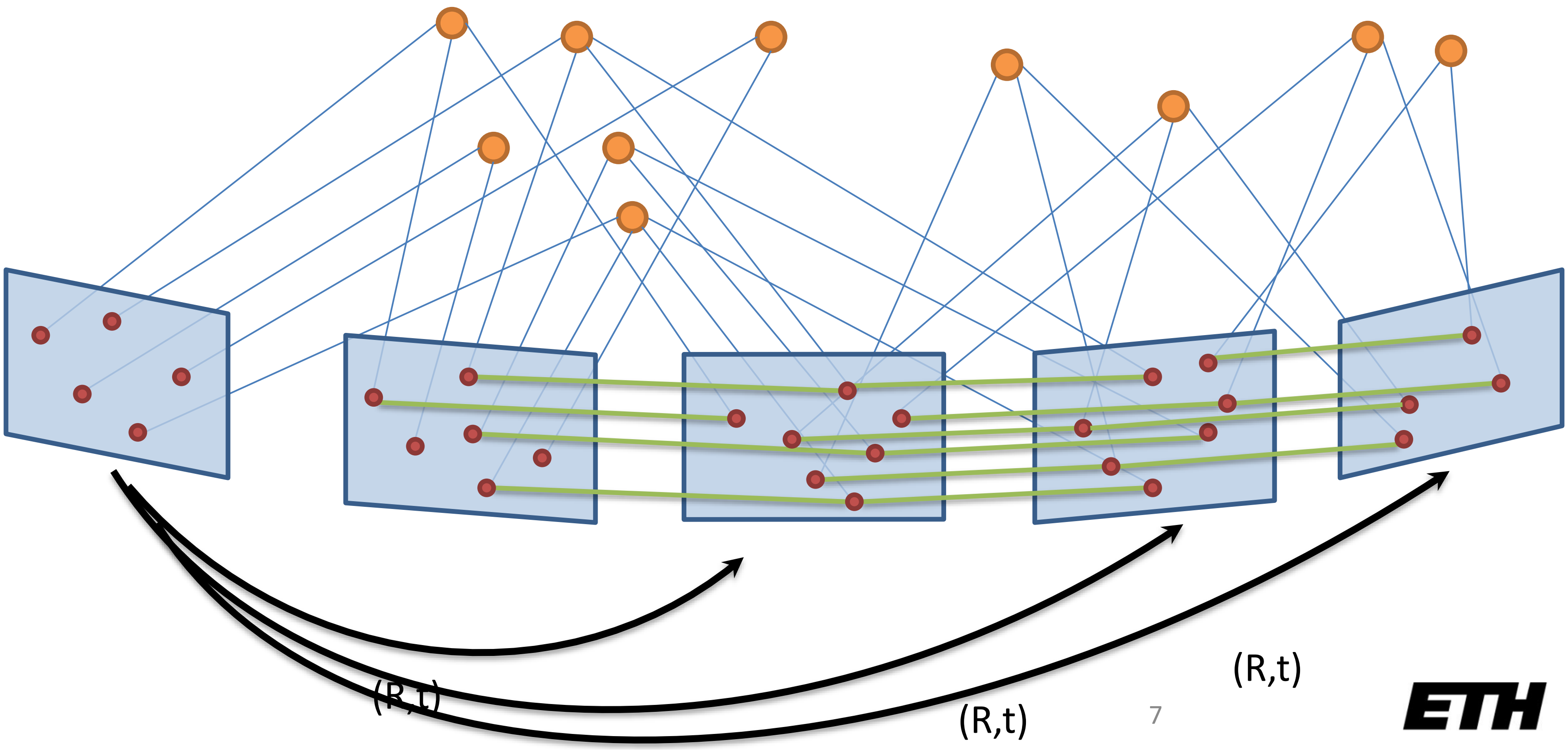
- Triangulation: 3D Points



E → (R,t)

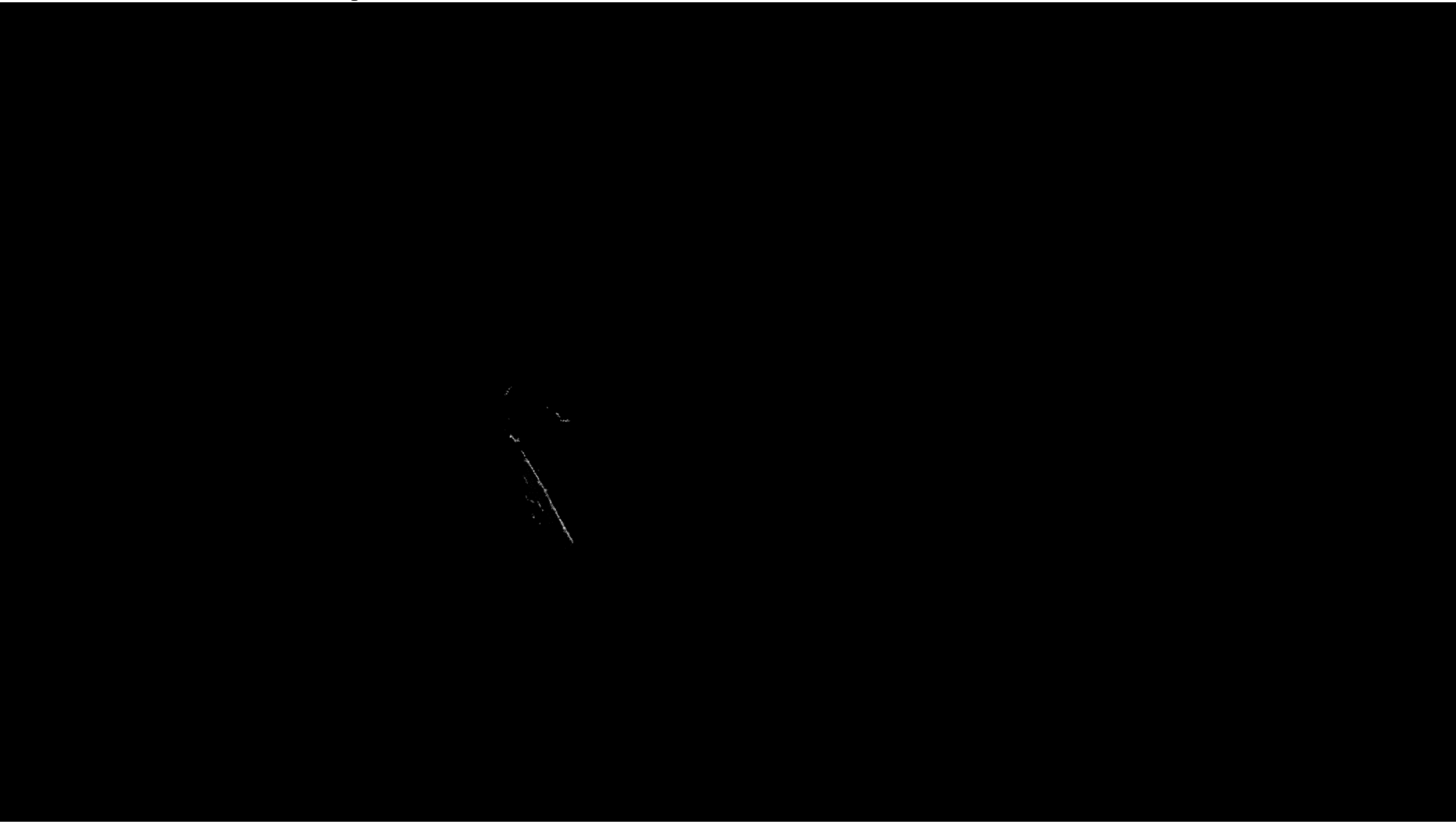
Recap: Structure-From-Motion

- Subsequent views: Perspective pose estimation



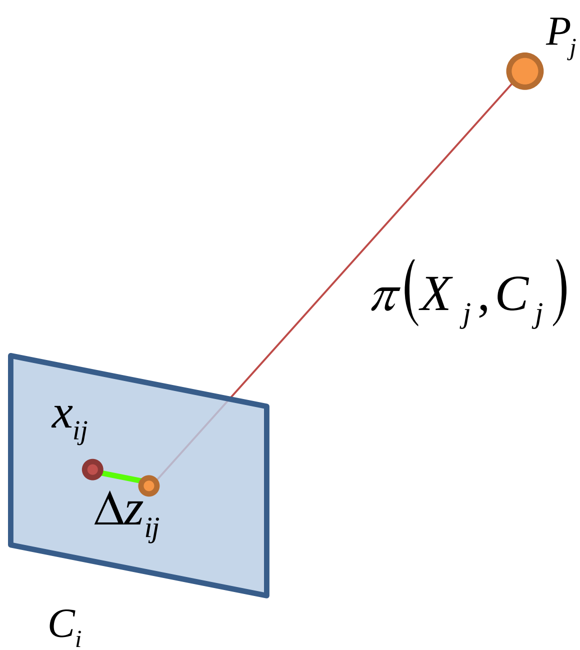
ETH

Recap: Structure-From-Motion



Bundle Adjustment

- Refinement step in Structure-from-Motion.
- Refine a visual reconstruction to produce jointly optimal 3D structures P and camera poses C.
- Minimize total re-projection errors Δz.



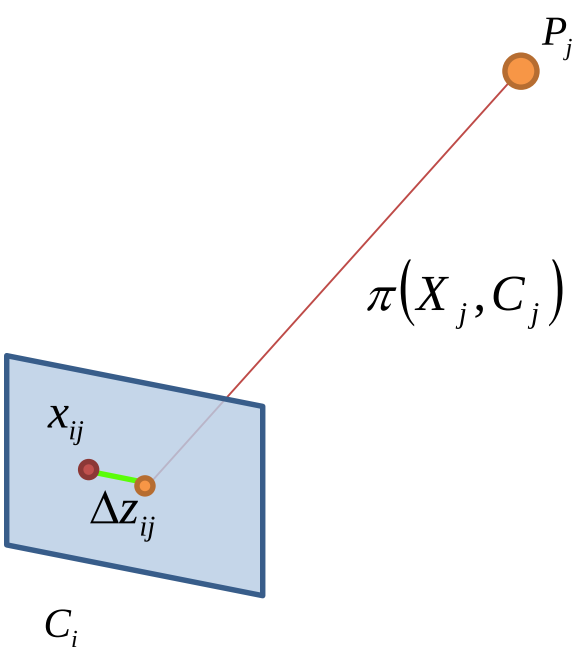
Cost Function:

$$\underset{X}{\operatorname{argmin}} \sum_i \sum_j \left\| \underbrace{x_{ij} - \pi(P_j, C_i)}_{\Delta z_{ij}} \right\|_{W_{ij}}^2$$

$$X = [P, C]$$

Bundle Adjustment

- Refinement step in Structure-from-Motion.
- Refine a visual reconstruction to produce jointly optimal 3D structures P and camera poses C.
- Minimize total re-projection errors Δz.



Cost Function:

$$\underset{X}{\operatorname{argmin}} \underbrace{\sum_i \sum_j \Delta z_{ij}^T W_{ij} \Delta z_{ij}}_{f(X)}$$

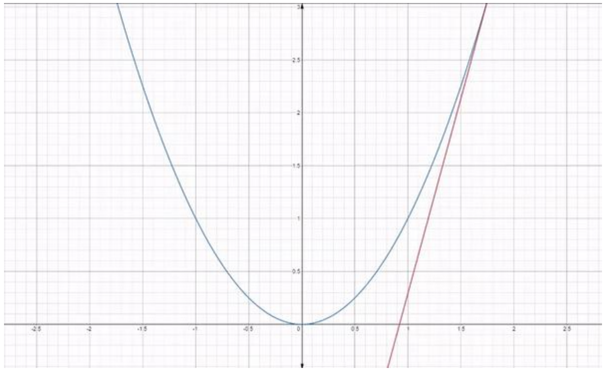
W_{ij}^{-1} : Measurement error covariance

$$X = [P, C]$$

Bundle Adjustment

- Minimize the cost function: $\underset{X}{\operatorname{argmin}}\, f(X)$
  1. Gradient Descent
  2. Newton Method
  3. Gauss-Newton
  4. Levenberg-Marquardt

Bundle Adjustment



1. Gradient Descent

Initialization: $X_k = X_0$

Iterate until convergence

Compute gradient: $g = \left.\dfrac{\partial f(X)}{\partial X}\right|_{X=X_k} = \Delta Z^T W J$

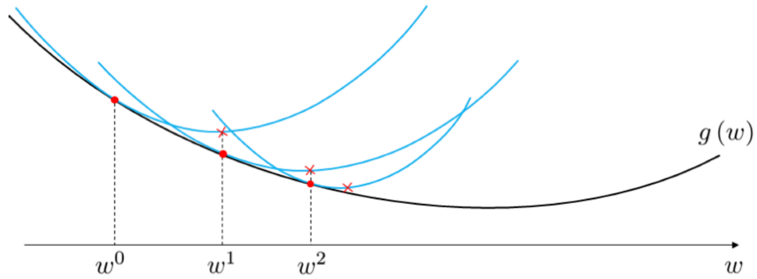
Update: $X_k \leftarrow X_k - \eta g$

$J = \dfrac{\partial \pi}{\partial X}$: Jacobian

η : Step size

Slow convergence near minimum point!

Bundle Adjustment



2. Newton Method

2nd order approximation (Quadratic Taylor Expansion):

$$f(X+\delta)\big|_{X=X_K} \approx f(X) + g\delta + \tfrac{1}{2}\delta^T H\delta\big|_{X=X_K}$$

Hessian matrix $: H = \left.\dfrac{\partial^2 f(X+\delta)}{\partial\delta^2}\right|_{X=X_k}$

Find δ that minimizes $f(X+\delta)\big|_{X=X_K}$!

Bundle Adjustment

2. Newton Method

Differentiate and set to 0 gives:

$$\delta = -H^{-1} g$$

Update: $X_k \leftarrow X_k + \delta$

Computation of H is not trivial and might get stuck at saddle point!

Bundle Adjustment

3. Gauss-Newton

$$H = J^T W J + \cancel{\sum_i \sum_j \Delta Z_{ij} W_{ij} \frac{\partial^2 \pi_{ij}}{\partial X^2}}$$

$$\downarrow$$

$$H \approx J^T W J$$

Normal equation:

$$J^T W J \delta = -J^T W \Delta Z$$

Update: $X_k \leftarrow X_k + \delta$

Might get stuck and slow convergence at saddle point!

ETH

Bundle Adjustment

4. Levenberg-Marquardt

Regularized Gauss-Newton with damping factor λ .

$$\underbrace{\left(J^T W J + \lambda I\right)}_{H_{LM}} \delta = -J^T W \Delta Z$$

$\lambda \rightarrow 0$: Gauss-Newton (when convergence is rapid)

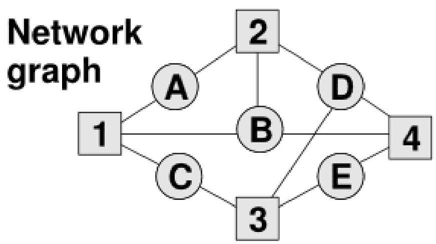
$\lambda \rightarrow \infty$: Gradient descent (when convergence is slow)

Adapt λ during optimization:

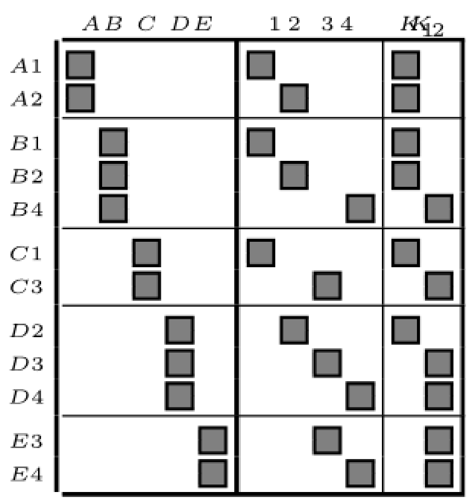
- Decrease λ when function value decreases
- Increase λ otherwise

Structure of the Jacobian and Hessian Matrices

- Sparse matrices since 3D structures are locally observed.



J =

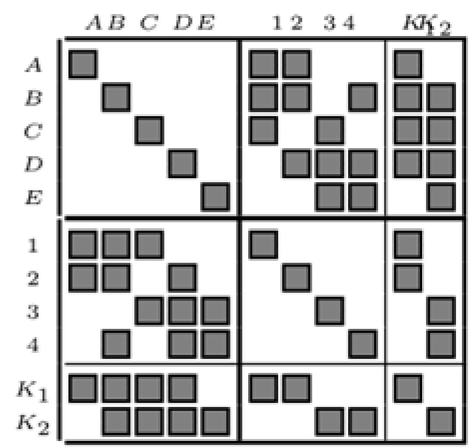


Efficiently Solving the Normal Equation

- Schur Complement: Exploit structure of H

$$H_{LM}\delta = -J^T W \Delta Z$$

$H_{LM} =$



3D Structures

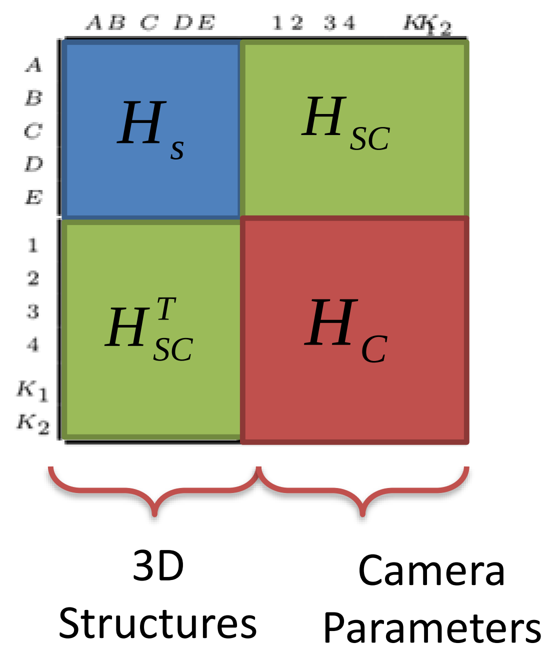
Camera Parameters

Efficiently Solving the Normal Equation

- Schur Complement: Exploit structure of H

$$H_{LM}\delta = -J^{T}W\Delta Z$$

$$H_{LM} =$$



Efficiently Solving the Normal Equation

- Schur Complement: Obtain reduced system

$$H_{LM}\delta = -J^T W \Delta Z$$

$$\begin{bmatrix} H_S & H_{SC} \\ H_{SC}^T & H_C \end{bmatrix}\begin{bmatrix} \delta_S \\ \delta_C \end{bmatrix} = \begin{bmatrix} \varepsilon_S \\ \varepsilon_C \end{bmatrix} \begin{array}{l} \leftarrow \text{3D Structures} \\ \leftarrow \text{Camera Parameters} \end{array}$$

Multiply both sides by: $\begin{bmatrix} I & 0 \\ -H_{SC}^T H_S^{-1} & I \end{bmatrix}$

$$\begin{bmatrix} H_S & H_{SC} \\ 0 & H_C - H_{SC}^T H_S^{-1} H_{SC} \end{bmatrix}\begin{bmatrix} \delta_S \\ \delta_C \end{bmatrix} = \begin{bmatrix} \varepsilon_S \\ \varepsilon_C - \varepsilon_S H_{SC}^T H_S^{-1} \end{bmatrix}$$

Efficiently Solving the Normal Equation

- Schur Complement: Obtain reduced system

$$\begin{bmatrix} H_S & H_{SC} \\ 0 & H_C - H_{SC}^T H_S^{-1} H_{SC} \end{bmatrix} \begin{bmatrix} \delta_S \\ \delta_C \end{bmatrix} = \begin{bmatrix} \varepsilon_S \\ \varepsilon_C - \varepsilon_S H_{SC}^T H_S^{-1} \end{bmatrix}$$

First solve for δ_C from:

Easy to invert a block diagonal matrix

$$\underbrace{(H_C - H_{SC}^T H_S^{-1} H_{SC})}_{\text{Schur Complement}}\delta_C = \varepsilon_C - \varepsilon_S H_{SC}^T H_S^{-1}$$

Schur Complement
(Sparse and Symmetric Positive Definite Matrix)

Solve for δ_{SC} by backward substitution.

ETH

Efficiently Solving the Normal Equation

$$(H_C - H_{SC}^T H_S^{-1} H_{SC})\delta_C = \varepsilon_C - \varepsilon_S H_{SC}^T H_S^{-1} \quad \equiv \quad Ax = b$$

Don't solve as $x=A^{-1}b$: A is sparse, but A^{-1} is not!

- Use sparse matrix factorization to solve system
  1. LU Factorization $\longrightarrow$ $A = LU$
  2. QR factorization $\longrightarrow$ $A = QR$
  3. Cholesky Factorization $\longrightarrow$ $A = LL^T$

Solve for x by forward-backward substitutions

- Iterative methods
  1. Conjugate gradient
  2. Gauss-Seidel

Robust Cost Function

- Non-linear least squares: $\underset{X}{\operatorname{argmin}} \sum_{ij} \Delta z_{ij}^T W_{ij} \Delta z_{ij}$
- Maximum log-likelihood solution:

$$\underset{X}{\operatorname{argmin}} - \ln p(Z \mid X)$$

- Assume that:
  1. X is a random variable that follows Gaussian distribution.
  2. All observations are independent.

$$\underset{X}{\operatorname{argmin}} - \ln p(X \mid Z) = \underset{X}{\operatorname{argmin}} - \ln \left\{ \prod_{ij} c_{ij} \exp\left(- \Delta z_{ij}^T W_{ij} \Delta z_{ij} \right) \right\}$$

$$= \underset{X}{\operatorname{argmin}} \sum_{ij} \Delta z_{ij}^T W_{ij} \Delta z_{ij}$$

Robust Cost Function

- Gaussian distribution assumption is not true in the presence of outliers!
- Causes wrong convergences.

Robust Cost Function

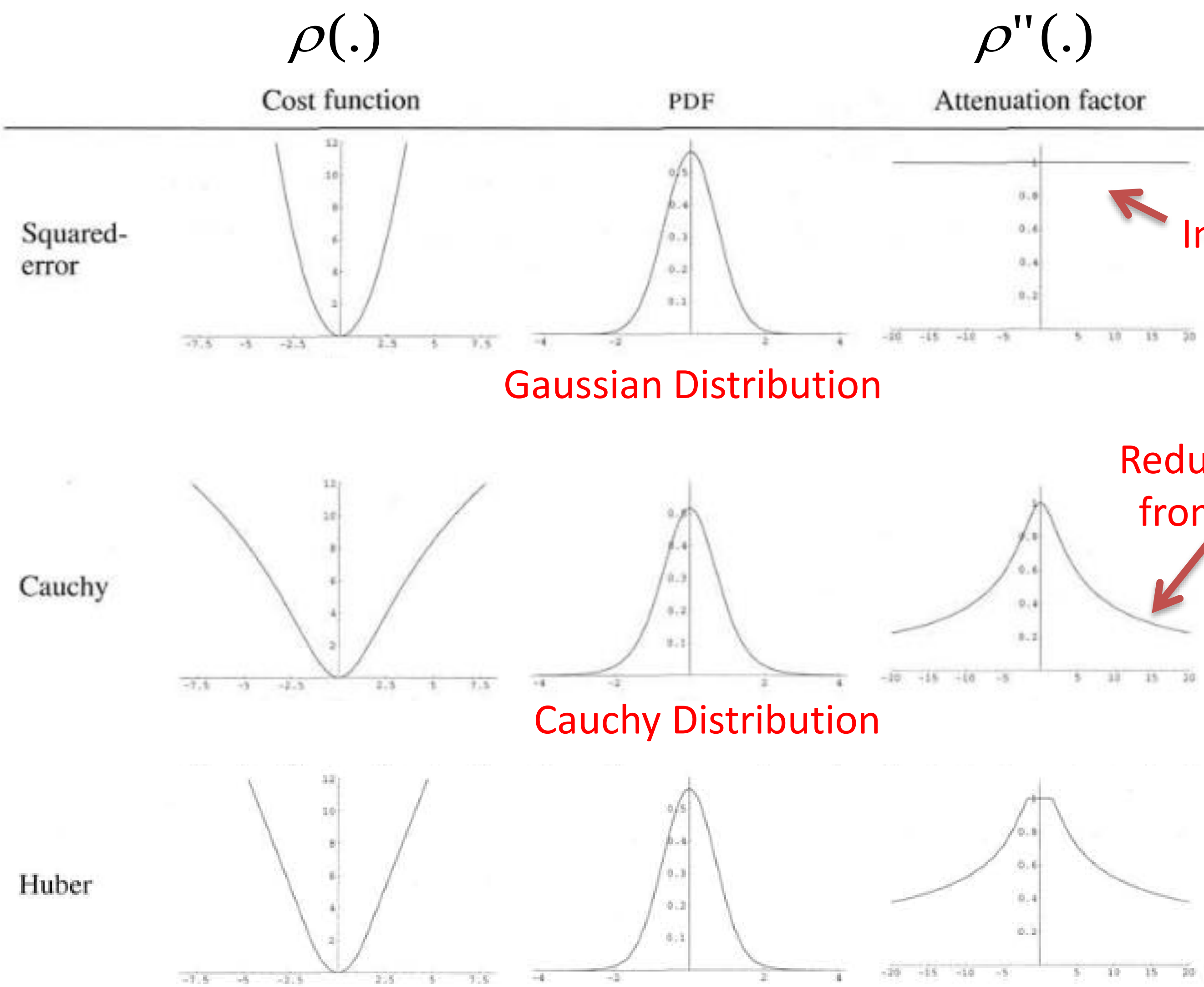
$$\underset{X}{\operatorname{argmin}} \sum_{ij} \rho_{ij}\left(\Delta z_{ij}\right) \equiv \underset{X}{\operatorname{argmin}} \sum_{ij} \Delta z_{ij}^{T} S_{ij} \Delta z_{ij}$$

Robust Cost Function

W_{ij} scaled with ρ''_{ij}

- Similar to iteratively re-weighted least-squares.
- Weight is iteratively rescaled with the attenuating factor ρ''_{ij} .
- Attenuating factor is computed based on current error.

Robust Cost Function



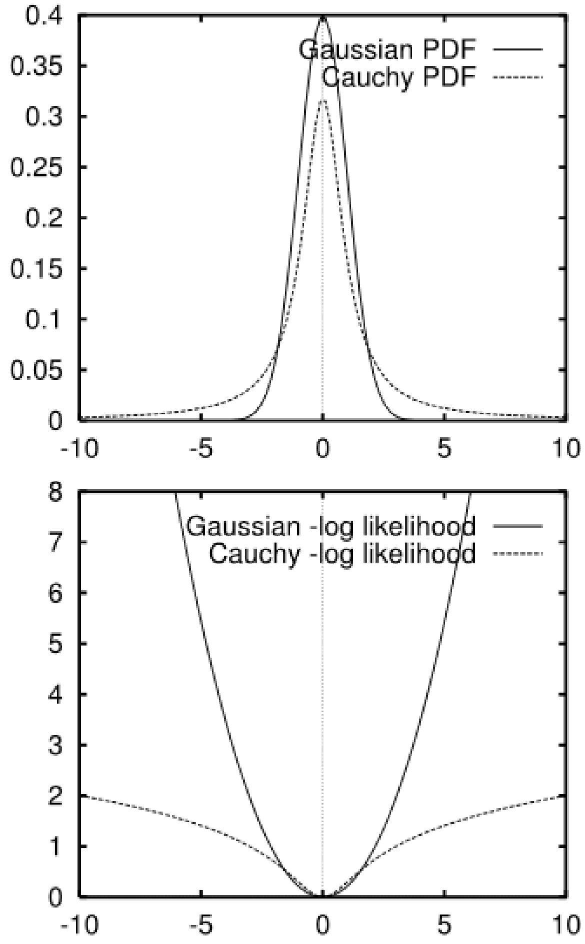
Influence from high errors

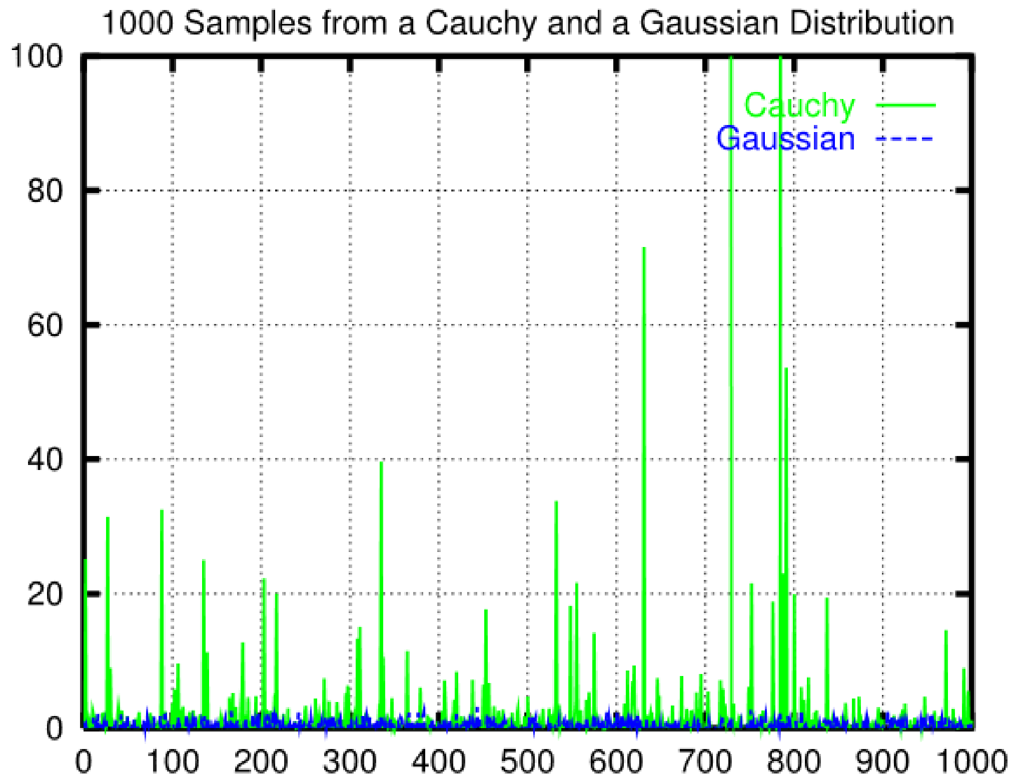
Gaussian Distribution

Reduced influence from high errors

Cauchy Distribution

Robust Cost Function





Outliers are taken into account in Cauchy!

State-of-the-Art Solvers

- Google Ceres:
  - https://code.google.com/p/ceres-solver/
- g2o:
  - https://openslam.org/g2o.html
- GTSAM:
  - https://collab.cc.gatech.edu/borg/gtsam/
- Multicore Bundle Adjustment
  - http://grail.cs.washington.edu/projects/mcba/

ETH

Lecture Overview

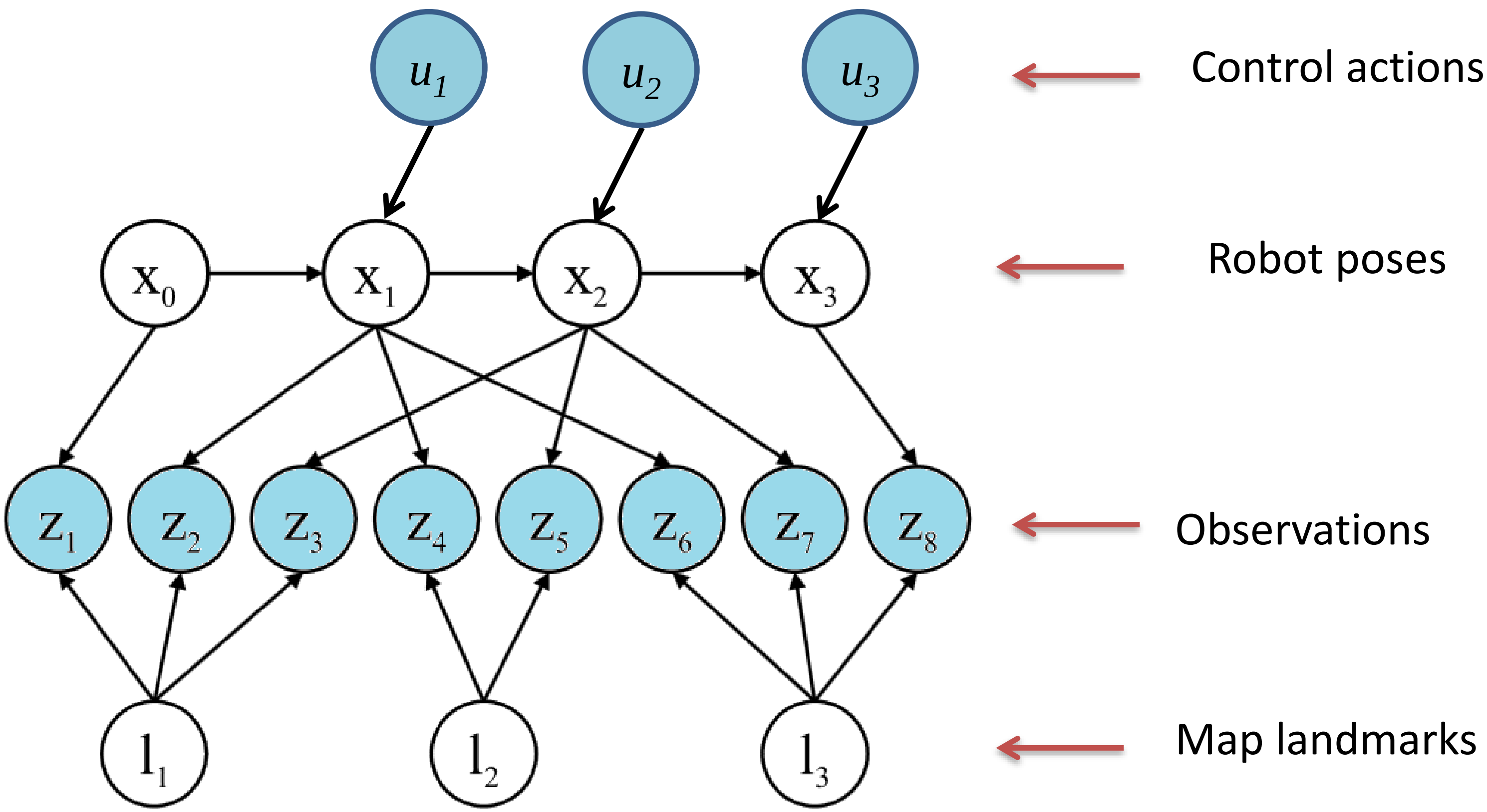
- Bundle Adjustment in Structure-from-Motion

- Simultaneous Localization & Mapping (SLAM)

Simultaneous Localization & Mapping (SLAM)

- Robot navigates in unknown environment:
  - Estimate its own pose
  - Acquire a map model of its environment.
- Chicken-and-Egg problem:
  - Map is needed for localization (pose estimation).
  - Pose is needed for mapping.
- Highly related to Structure-From-Motion.

Full SLAM: Problem Definition



Full SLAM: Problem Definition

- Maximum a Posteriori (MAP) solution:

$$\operatorname*{argmax}_{X,L} p\left(X, L \mid Z, U\right) = \operatorname*{argmax}_{X,L} p(X_0) \prod_{i=1}^{M} p(x_i \mid x_{i-1}, u_i) \prod_{k=1}^{K} p(z_k \mid x_{ik}, l_{jk})$$

Full SLAM

$$\underset{X,L}{\operatorname{argmax}}\, p\left(X, L \mid Z, U\right) = \underset{X,L}{\operatorname{argmax}}\, p(X_0) \prod_{i=1}^{M} p(x_i \mid x_{i-1}, u_i) \prod_{k=1}^{K} p(z_k \mid x_{ik}, l_{jk})$$

Negative log-likelihood →

$$= \underset{X,L}{\operatorname{argmin}} \left\{ -\sum_{i=1}^{M} \ln p(x_i \mid x_{i-1}, u_i) - \sum_{k=1}^{K} \ln p(z_k \mid x_{ik}, l_{jk}) \right\}$$

Likelihoods:

$$p(x_i \mid x_{i-1}, u_i) \propto \exp\{-\| f(x_{i-1}, u) - x_i \|_{\Lambda_i}^2 \}$$

Process model

$$p(z_k \mid x_{ik}, l_{jk}) \propto \exp\{-\| h(x_{ik}, l_{jk}) - z_k \|_{\Sigma_k}^2 \}$$

Measurement model

Full SLAM

$$\underset{X,L}{\operatorname{argmax}}\, p(X,L \mid Z,U) = \underset{X,L}{\operatorname{argmin}} \left\{ -\sum_{i=1}^{M} \ln p(x_i \mid x_{i-1}, u_i) - \sum_{k=1}^{K} \ln p(z_k \mid x_{ik}, l_{ik}) \right\}$$

Putting the likelihoods into the equation:

$$\underset{X,L}{\operatorname{argmax}}\, p(X,L \mid Z,U) = \underset{X,L}{\operatorname{argmin}} \left\{ \sum_{i=1}^{M} \left\| f(x_{i-1}, u_i) - x_i \right\|_{\Lambda_i}^2 + \sum_{k=1}^{K} \left\| h(x_{ik}, l_{ik}) - z_k \right\|_{\Sigma_k}^2 \right\}$$

Minimization can be done with Levenberg-Marquardt (similar to bundle adjustment problem)!

ETH

Full SLAM

Normal Equations:

Weight made up of Λ_i , Σ_k

$$\left(J^T W J + \lambda I\right)\delta = -J^T W \Delta Z$$

Jacobian made up of $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial u}$, $\frac{\partial h}{\partial x}$, $\frac{\partial h}{\partial l}$

Can be solved with sparse matrix factorization or iterative methods

Solving the full SLAM problem rather expensive for larger scenes

Online SLAM: Problem Definition

- Estimate current pose x_t and full map L:

$$p(x_t, L \mid Z, U) = \underbrace{\int\int\ldots\int p(X, L \mid Z, U)\, dx_1\, dx_2 \ldots dx_{t-1}}_{\text{Previous poses are marginalized out}}$$

- Inference with:
  1. (Extended) Kalman Filter (EKF SLAM)
  2. Particle Filter (FastSLAM)

EKF SLAM

- Assumes: pose x_t and map L are random variables that follow Gaussian distributions.
- Hence,

$$p(x_t, L \mid Z, U) \sim \mathrm{N}(\mu, \Sigma)$$

Mean   Error covariance

- (Extended) Kalman Filter iteratively
  - Predicts pose & map based on process model
  - Corrects prediction based on observations

ETH

EKF SLAM

Prediction:

$$\bar{\mu}_t = f(u_t, \mu_{t-1})$$ Process model

$$\bar{\Sigma}_t = F_t \Sigma_{t-1} F_t^T + R_t$$ Error propagation with process noise

Correction:

$$y_t = z_t - h(\bar{\mu}_t)$$ Measurement residual (innovation)

$$K_t = \bar{\Sigma}_t H_t^T (H_t \bar{\Sigma}_t H_t^T + Q_t)^{-1}$$ Kalman gain

$$\mu_t = \bar{\mu}_t + K_t y_t$$ Update mean

$$\Sigma_t = (I - K_t H_t)\bar{\Sigma}_t$$ Update covariance

Measurement Jacobian $H_t = \dfrac{\partial h(\bar{\mu}_t)}{\partial x_t}$ Process Jacobian $F_t = \dfrac{\partial f(u_t, \mu_{t-1})}{\partial x_{t-1}}$

ETH

Structure of Mean and Covariance

$$\mu_t = \begin{pmatrix} x \\ y \\ \theta \\ l_1 \\ l_2 \\ \vdots \\ l_N \end{pmatrix}, \quad \Sigma_t = \begin{pmatrix} \sigma_x^2 & \sigma_{xy} & \sigma_{x\theta} & \sigma_{xl_1} & \sigma_{xl_2} & \cdots & \sigma_{xl_N} \\ \sigma_{xy} & \sigma_y^2 & \sigma_{y\theta} & \sigma_{yl_1} & \sigma_{yl_2} & \cdots & \sigma_{yl_N} \\ \sigma_{x\theta} & \sigma_{y\theta} & \sigma_\theta^2 & \sigma_{\theta l_1} & \sigma_{\theta l_2} & \cdots & \sigma_{\theta l_N} \\ \sigma_{xl_1} & \sigma_{yl_1} & \sigma_{\theta l_1} & \sigma_{l_1}^2 & \sigma_{l_1 l_2} & \cdots & \sigma_{l_1 l_N} \\ \sigma_{xl_2} & \sigma_{yl_2} & \sigma_{\theta l_2} & \sigma_{l_1 l_2} & \sigma_{l_2}^2 & \cdots & \sigma_{l_2 l_N} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \sigma_{xl_N} & \sigma_{yl_N} & \sigma_{\theta l_N} & \sigma_{l_1 l_N} & \sigma_{l_2 l_N} & \cdots & \sigma_{l_N}^2 \end{pmatrix}$$

Covariance is a dense matrix that grows with increasing map features!

True robot and map states might not follow unimodal Gaussian distribution!

Particle Filtering: FastSLAM

- Particles represents samples from the posterior distribution $p(x_t, L \mid Z, U)$.
- $p(x_t, L \mid Z, U)$ can be any distribution (need not be Gaussian).

FastSLAM

Each particle represents:

$$p_t^m = \{x_t^m, < \mu_{1,t}^m, \Sigma_{1,t}^m >, \underbrace{< \mu_{2,t}^m, \Sigma_{2,t}^m >}_{\text{Landmark state (mean and covariance)}} \ldots < \mu_{N,t}^m, \Sigma_{N,t}^m >\}$$

Robot state

Landmark state
(mean and covariance)

$$x_t^m \sim p(x_t \mid x_{t-1}, u_t)$$

Sample the robot state from the process model

$$p(L_{n,t}^m \mid x_t^m, z_t)$$

N Kalman filter Landmark updates

$$w_t^m \propto p(z_t \mid L_t^m, x_t^m)$$

Weight update

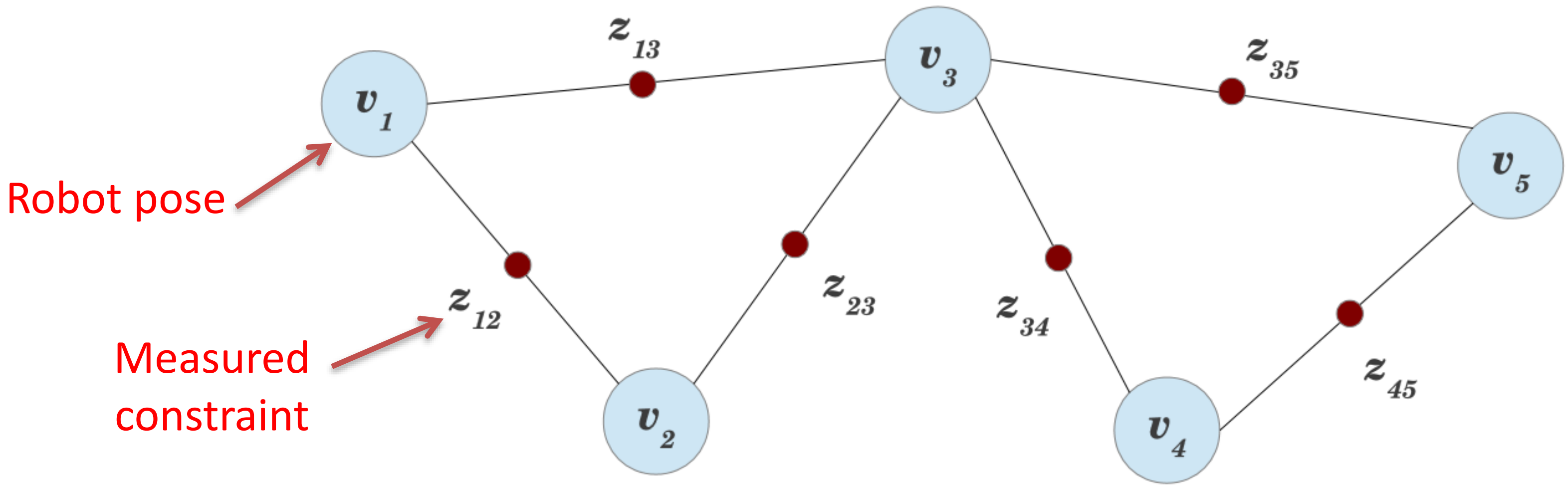
Resampling based on current state

ETH

FastSLAM

- Many particles needed for accurate results.
- Computationally expensive for high state dimensions.

Pose-Graph SLAM



- Constraints: Relative pose estimates from 3D structure.
- Don't update 3D structure (fixed wrt. to some pose).
- Optimizes poses as $\underset{X}{\operatorname{argmin}} \sum_{ij} \left\| z_{ij} - \underbrace{h(v_i, v_j)}_{\text{Relative transformation between poses}} \right\|_{\Sigma_{ij}}^2$
- Can be used to minimize loop-closure errors.

Summary

- Bundle Adjustment
  - Refine 3D points and poses in Structure-From-Motion.
  - Efficient computation by exploiting structure & sparsity.
  - Core step in every Structure-From-Motion (SFM) pipeline.
- Simultaneous Localization and Mapping
  - Very similar to Incremental SFM.
  - Typically includes some motion model.
  - Two general approaches to SLAM:
    - (Local) Bundle Adjustment (not discussed in lecture)
    - Filter-based techniques (EKF SLAM, FastSLAM)
  - Pose-Graph SLAM (loop-closure handling)

Schedule

Feb 19	Introduction
Feb 26	Geometry, Camera Model, Calibration
Mar 4	Guest lecture + Features, Tracking / Matching
Mar 11	**Project Proposals by Students**
Mar 18	3DV conference
Mar 25	Structure from Motion (SfM) + papers
Apr 1	Easter break
Apr 8	Dense Correspondence (stereo / optical flow) + papers
Apr 15	**Bundle Adjustment & SLAM + papers**
Apr 22	**Student Midterm Presentations**
Apr 29	Multi-View Stereo & Volumetric Modeling + papers
May 6	3D Modeling with Depth Sensors + papers
May 13	Guest lecture + papers
May 20	Holiday

Next week: Midterm Presentations

Reminder:
Prepare short presentation (3-5min) for Monday!